A Novel Nonlinear Vector Filter for Grey-Level Image Filtering

Kaijun Tang, Moncef Gabbouj and Jaakko Astola
Signal Processing Laboratory
Tampere University of Technology
P.O.Box 553, FIN-33101 Tampere, Finland

ABSTRACT — **In this paper, a novel nonlinear vector filtering algorithm is developed with a particular application to grey-level images. In the algorithm, two-dimensional image data are divided into blocks, and the data in each block form a vector sample. Then, the nonlinear vector filtering is performed over the signals of vector sample in overlapping or non-overlapping blocks. Results show that this nonlinear vector filter has excellent noise attenuation and edge preservation properties when it is applied for grey-level images.**

1 Introduction

Until recently, little attention was paid to the joint processing of vector signals. Vector median filter is one of typical nonlinear vector filters [1], [2]. By definition, the vector median is one of samples which minimizes a sum of distances to other samples. As a result, the vector median filter can not achieve the same noise attenuation as the componentwise median filter due to fewer sample candidates. However, if one takes a vector sample not necessarily in the window minimizing the sum of distances, better performance is expected to be achieved [3]. This filter is called the nonlinear vector filter in this paper.

In grey-level image processing, the median filter is an effective tool in removing impulse-type noise without blurring edges of images. However, it has poor performance in attenuating medium/short-tailed noise, e.g., white Gaussian and uniform noises. On the other hand, the mean filter and midpoint filter are proven to be optimal in suppressing these noises, respectively, but they is not applied to image processing because of blurring edges of images. In this paper, we propose the application of the nonlinear vector filter to grey-level image processing by constructing block vectors. Since the output of the nonlinear vector filter is not necessarily one of input samples, the effects of streaking and blotching are reduced. Another property of the nonlinear vector filter is excellent edge response [3].

2 Nonlinear Vector Filtering Algorithm

Given a set of N vector samples $\mathbf{X}(1)$, $\mathbf{X}(2)$, $\cdots$, $\mathbf{X}(N)$, the output of the nonlinear vector filter is given by $\mathbf{X}$ minimizing a function of weighted sum of distances,

$$\mathbf{X} = \min_{\mathbf{X}} f(\mathbf{X}) \tag{1}$$

with

$$f(\mathbf{X}) = \sum_{i=1}^{N} W_i \|\mathbf{X} - \mathbf{X}(i)\| \tag{2}$$

where $\|\cdot\|$ denotes L_2 norm. W_i, $i = 1, 2, \cdots, N$, are weights corresponding to $\mathbf{X}(i)$. Unless stated otherwise, If $\mathbf{X}$ minimizing $f(\mathbf{X})$ is constrained to be one of the N samples, it is reduced to weighted form of the vector median [2]. In general, a closed-form solution for $\mathbf{X}$ does not exist. An estimate is achieved by computing iteratively the gradient of the weighted sum of distances as following,

$$\mathbf{X}_{k+1} = \mathbf{X}_k - \mu \nabla_k f(\mathbf{X}_k) \tag{3}$$

where k is the step or iteration number and μ is step size.

In the case of the L_2 norm, the gradient function is computed as follows,

$$\nabla_k f = \begin{bmatrix} \sum_{i=1}^{N} W_i \frac{x_k^{(1)} - x^{(1)}(i)}{\|\mathbf{X}_k - \mathbf{X}(i)\|} \\ \sum_{i=1}^{N} W_i \frac{x_k^{(2)} - x^{(2)}(i)}{\|\mathbf{X}_k - \mathbf{X}(i)\|} \\ \vdots \\ \sum_{i=1}^{N} W_i \frac{x_k^{(p)} - x^{(p)}(i)}{\|\mathbf{X}_k - \mathbf{X}(i)\|} \end{bmatrix} \tag{4}$$

where $x_k^{(j)}$ and $x^{(j)}(i)$ denote the j-th component of $\mathbf{X}_k$ and $\mathbf{X}(i)$, respectively.

In order to guarantee the existence of the gradient function, $\mathbf{X}_k - \mathbf{X}(i) \neq 0$ should hold for every k and i. If $\mathbf{X}_k$ is equal to $\mathbf{X}(i)$, $\|\mathbf{X}_k - \mathbf{X}(i)\|$ will be removed from the weighted sum terms. In the case that the

TABLE I. OUTPUT MEAN SQUARE ERRORS OF VECTOR FILTERS

Multivariate Filter	*output mean square errors*		
	Biexponential	Gaussian	Uniform
Mean Filter	0.1411	0.1409	0.1495
Componentwise Median Filter	0.1160	0.2057	0.3427
Vector Median Filter	0.2194	0.3393	0.5129
Nonlinear Vector Filter	0.1040	0.1464	0.1920

solution is not unique, the multivariate sample closest to the center of the window is preferred to others. Therefore, the center sample within the window is selected as an initial value of the gradient search . The choice of the step size is determined empirically. A small step size leads to relatively slow convergence rate. A large step size implies fast convergence rate but leads to instability of the gradient search. Usually, it is difficult to evaluate the bound of the step size for the stability of the gradient search algorithm. Table I shows the average noise attenuation performance of several nonlinear filters including the mean filter over i.i.d. three-component random vector samples under different noise distributions, where all vector noises have zero mean unit variance in each of three components and the length of the filters is 7. The nonlinear vector filter is found to have better noise attenuation than the componentwise median filter, This implies that the joint filtering of compounds outperforms independent one.

To apply the nonlinear vector filtering defined by equation (1) and (2) to grey-level image processing, we consider a two-dimensional image field. In fig. 1(a), a $p \times N$ nonlinear vector filter window is designed as shown by dotted lines. The window contains N blocks having the size of p rows $\times$ 1 column. In the blocks, p samples consist of a vector-valued sample. For example, $x(i,j), i = 1, 2, \cdots, p$ form $\mathbf{X}(j)$, a p-component vector sample and $x(i,j)$, pixel sample is treated as th-i component of $\mathbf{X}(j)$. Thus, the N pixels on the same row belong to the same component of the vector samples. As the result of the nonlinear vector filtering, we get the output of p samples as the $p \times N$ window moves by one pixel along the horizontal direction. Thus, the nonlinear vector filter will move to next p lines after it performs filtering over the present p lines. In order to preserve details of images, we could also have the N pixels in the same component to form a cross-shape structure. For example, we form a 3×3 cross window in which the all pixels consist of a vector sample, and they are indexed as the componets of the vector. Then, we generate other four vector samples the same way by moving the 3×3 window horizontally and vertically by one pixel so that the same components of the vector samples all form 3×3 cross structure as shown in Fig. 1(b).

As a comparison of the performance of the nonlinear vector filter and other linear and nonlinear filters in grey-level image processing, Figs. 2 demonstrate the filtering results and the output variances of the average filter, the median filter and the nonlinear vector filter when they are applied to grey-level image "peppers", corrupted by i.i.d. white Gaussian noise and uniform noise of variance 400, respectively. For the sake of comparison, the average filter and the median filter are p parallel 1-D filters having length of N. p and N are equal to 8 and 7, respectively. The average filter attenuates noise effectively but blurs edges too. The nonlinear vector filter gives better performance improvement than the median filter. For the purposes of detail preservation, we also constructed vector samples whose same components form a cross-shape structure shown by Fig. 3. Similar results followed.

3 Conclusion

In this paper, we applied the nonlinear vector filter for multivariate signal processing to grey-level images processing. For this purpose, vector samples are formed by pixels in neighboring blocks. The results of application to grey-level images demonstrate that the nonlinear vector filter outperforms the median filter in grey-level image processing.

References

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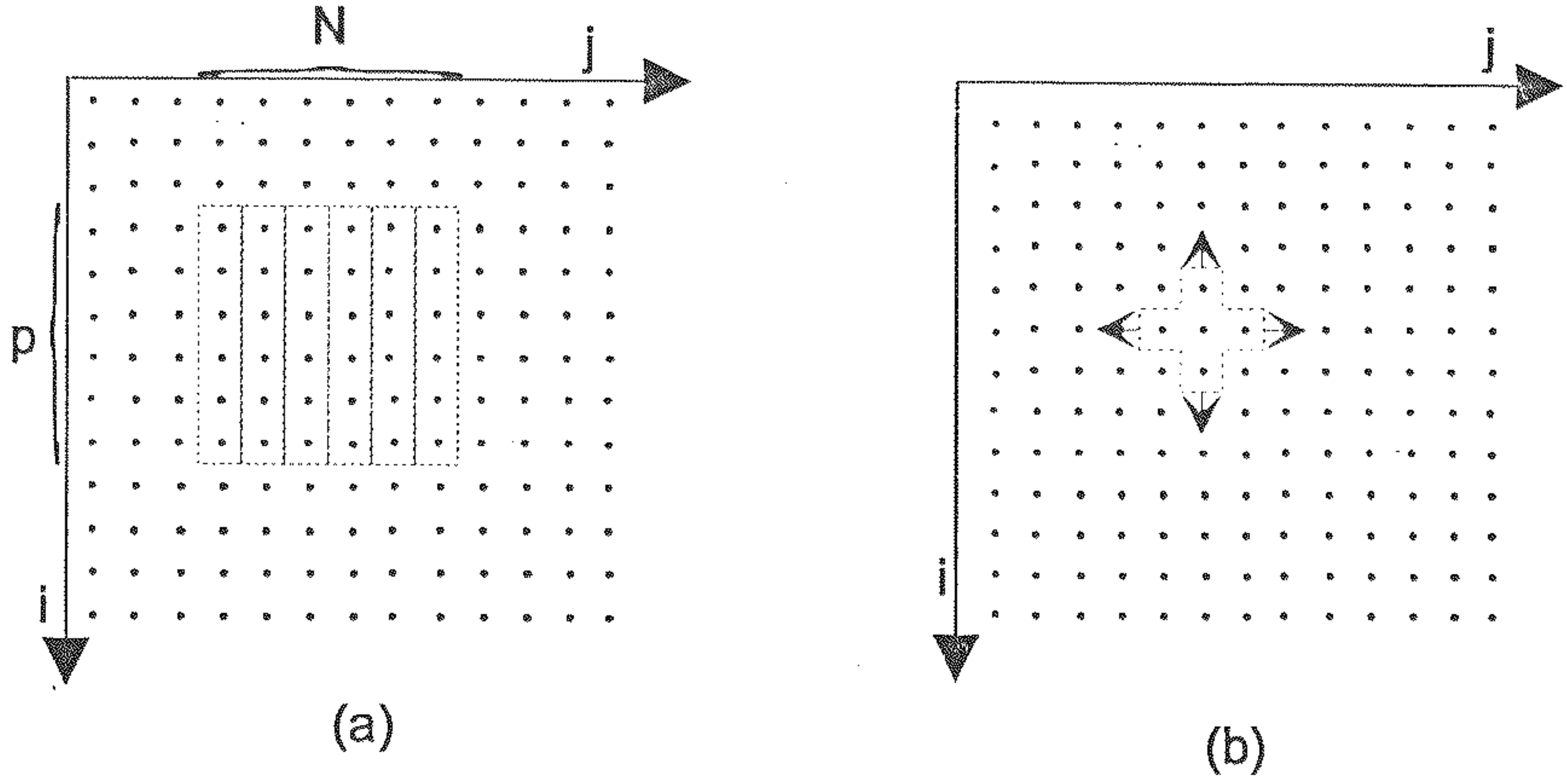
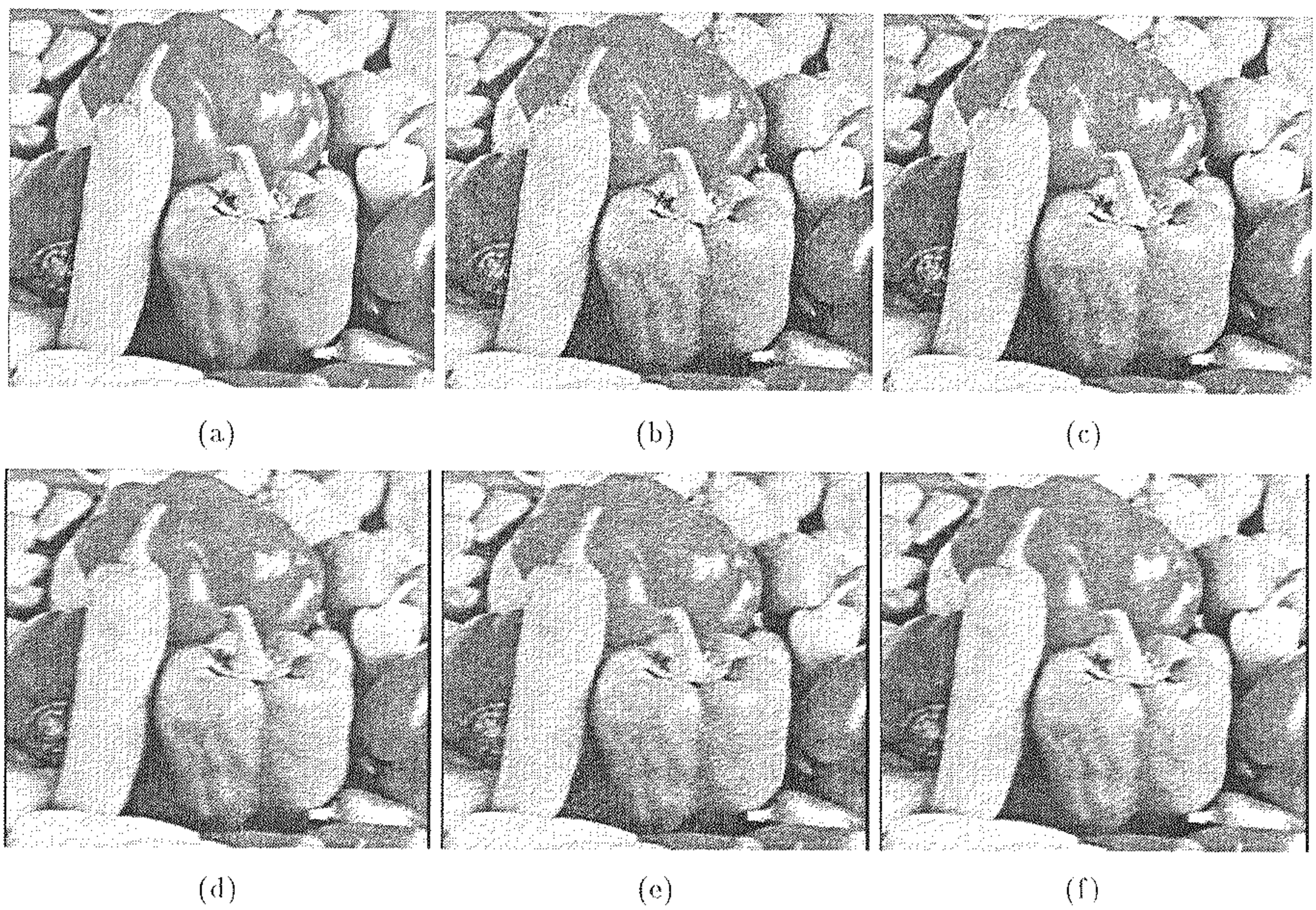


Fig.1 (a) $p \times N$ window for constructing N p-component vector samples. (b) 3×3 cross window for constructing a 5-component vector sample.



(g) (h) (i)

Fig. 2(a): 512 × 512 original image. (b): image corrupted by Gaussian noise of variance 400. (c): image corrupted by uniform noise of variance 400. (d): average filtering of (b) (σ^2=105.9). (e): median filtering of (b) ($\sigma^2 = 122.1$). (f): nonlinear vector filtering of (b)($\sigma^2 = 91.9$). (g): average filtering of (c) ($\sigma^2 = 106.4$). (h): median filtering of (c) ($\sigma^2 = 162.9$). (i): nonlinear vector filtering of (c) ($\sigma^2 = 98.1$).