Performance of Detail-Preserving Weighted Median Filters for Image Processing

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Abstract: In this paper, we present a method to design optimal two-dimensional detail-preserving weighted median (WM) filters in the sense that these filters preserve some desired image details, such as lines, corners, etc., under which they achieve the maximum noise attenuation. The performance of these detail-preserving WM filters is compared to that of FIR-median (FMH) hybrid filters which have shown good detail-preserving properties for image processing. Responses under different signal-to-noise ratios are carried out to characterize the performance of these detail-preserving WM filters.

1 Introduction

Median filters that incorporate rank-order operations have proven to be particularly effective in image restoration. This is because these filters can preserve or reconstruct edges, lines, and other image details while also removing noise or other processes which have altered the image. Median filters, however, often blur images when window becomes larger. Weighted median filters are, then, proposed to solve this problem [2]. Weighted median filters, when properly designed, can preserve finer image details than the standard median filter under the same noise attenuation. On the other hand, there may be a plenty of WM filters which preserve the desired image details. The problem, so called optimal filtering under structural constraints [3, 4], becomes to design optimal WM filters which have "best" noise reduction capability and at same time preserve desired image details. Several research works have been reported to obtain optimal WM filters, through training, under the MAE or the MSE error criterion [5, 6].

In this paper, we will consider nonadaptive detail-preserving WM filters. The principal advantage of nonadaptive filters is that neither *a priori* information of the image is required nor do local statistics need to be computed inside the filter's window. Thus, these nonadaptive filters have less computational complexity, and should be easier to implement than their adaptive counterparts. Using statistical properties of WM filters, we show that optimal detail-preserving WM filters can be found by minimizing the M_i's, a set of parameters which characterize the weights of a WM filter, under some pre-specified constraints on the weights. Then, the performance of these detail-preserving WM filters is compared to that of FIR-median hybrid filters, which have proven to be effective in preserving image details [1]. The performance of these detail-preserving WM filters is examined under different signal-to-noise (SNR) ratios.

In the next section, we shall give the definition of WM filters, and then introduce a set of parameters, called the M_i's, to characterize the weights of a WM filter. Design of these detail-preserving WM filters is discussed in Section 3. In Section 4, The performance of these detail-preserving WM filters and FMH filters is compared through test images. Responses under different signal-to-noise ratios are also presented in this section. Section 5 contains some conclusions.

2 Weighted Median filters

Definition 1: Let $\{X(\cdot,\cdot)\}$ and $\{Y(\cdot,\cdot)\}$ be the input image and the output image, respectively, of a WM filter with a $(2L+1)\times(2M+1)$ window

$$W = \{(s,t)| -L \le s \le L, -M \le t \le M\}.$$

Then the output of the WM filter with weight vector $\underline{W}$

$$\underline{W} = \{W_{s,t}|(s,t) \in W\} = (W_1, W_2, \cdots, W_N)$$

where $N = (2L+1)(2M+1)$, is given by

$$Y(i,j) = \text{MED}\{W_{s,t} \diamond X(i-s, j-t)|(s,t) \in W\} \quad (1)$$

where MED[·] denotes the median operation and $\diamond$ denotes duplication, i.e.

$$n \diamond X = \underbrace{X, ..., X}_{n\ times}.$$

Definition 2: Consider a WM filter with weight vector $\underline{W} = (W_1, W_2, ...W_N)$. Denote by $\mathcal{W}$ the multiset of weights, i.e.

$$\mathcal{W} = \{W_1, W_2, ..., W_N\}.$$

Denote by $\Upsilon^{[i]}$ the set of all submultisets of $\mathcal{W}$ having cardinality i, i.e.

$$\Upsilon^{[i]} = \{A | A \subseteq \mathcal{W}, |A| = i\}, \quad i = 0, 1, ..., N. \quad (2)$$

and by $\Omega^{[i]}$ the set of those subsets of $\Upsilon^{[i]}$ whose sum of elements is at least the threshold T, i.e.

$$\Omega^{[i]} = \{A | A \in \Upsilon^{[i]}, \sum_{W_j \in A} W_j \ge T\}, \quad (3)$$

such subsets are called *positive subsets.*
Definition 3: Define M_i the cardinality of set $\Omega^{[i]}$, i.e.

$$M_i = |\Omega^{[i]}|, \quad i = 0, 1, ...N. \quad (4)$$

Example 1: Given a WM filter with $\underline{W}$=(1, 4, 5, 3, 2), its threshold is $T = 8$. When $i = 1$, it is obvious that there is no such set which satisfies (3). When $i = 2$, there are two *positive subsets,* i.e.

$$\Omega^{[2]} = \{ \{3,5\}, \{4,5\}\}.$$

Similarly, one can find other positive subsets:

$$\Omega^{[3]} = \{\{1,3,4\}, \{2,3,4\}, \{1,2,5\}, \{1,3,5\}, \{2,3,5\}, \{1,4,5\}, \{2,4,5\}, \{3,4,5\}\},$$

$$\Omega^{[4]} = \{\{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\}\},$$

$$\Omega^{[5]} = \{ \{1,2,3,4,5\}\}.$$

By (4), we have $M_1 = 0$, $M_2 = 2$, $M_3 = 8$, $M_4 = 5$ and $M_5 = 1$.

3 Problem Formulation

Giving a set of detail-preserving requirements, e.g. lines, sharp corners, there may exist a number of WM filters preserving these structures. The designer's task is, then, to pick an WM filter among them in order to achieve the maximum noise attenuation. In the following, we shall formulate this problem when a constant signal is embedded in additive white noise.

Denote by $X(i,j)$ the input sample of a constant signal s corrupted by some additive white noise n to be filtered by a WM with weight vector $\underline{W}$. Then the output $\hat{s}$ of the WM filter, which is an estimate of s, can be written as:

$$\hat{s}(i,j) = \text{MED}\{W_{s,t} \diamond X(i-s, j-t)|(s,t) \in W\} \quad (5)$$

Take the mean square error (MSE) as the criterion function

$$J = E(s - \hat{s})^2. \quad (6)$$

According to statistical properties of WM filters [7], we have the following theorem.
Theorem: For WM filters with window size $N = 2K+1$, the mean square error J defined in (6) can be expressed as

$$J = \sigma_s{}^2 + \sum_{i=1}^{K} L_i M_i, \quad (7)$$

where ${\sigma_s}^2$ and L_i's are independent of the weights and

$$L_i \geq 0 \qquad i = 1, \cdots, K$$

From this theorem it is concluded that designing optimal WM filters under structural constraints is equivalent to minimizing the M_i's under detail-preserving requirements, i.e.

$$\begin{array}{c} \text{Minimize} \quad \sum_{i=1}^{K} L_i M_i \\ \text{subject to} \\ \text{structural constraints} \end{array} \tag{8}$$

Based on the idea above two-dimensional optimal WM filters were investigated under several typical structural constraints: line preservation, area preservation and compound details preservation. The weight vectors of the optimal WM filters were found [8], which are listed in Table 1. In the next section, we shall analysis the performance of these WM filters through test images.

Table 1: Optimal WM filters under several structural constraints

WM1	*Area Preservation* $M_1 = 0$ $M_2 = 0$ $M_3 = 0$ $M_4 = 28$	$\begin{pmatrix} 2 & 3 & 2 \\ 3 & 5 & 3 \\ 2 & 3 & 2 \end{pmatrix}$
WM2	*Horizontal Vertical Lines* $M_1 = 0$ $M_2 = 0$ $M_3 = 6$ $M_4 = 29$	$\begin{pmatrix} 1 & 3 & 1 \\ 3 & 5 & 3 \\ 1 & 3 & 1 \end{pmatrix}$
WM3	*Compound Preservation* $M_1 = 0$ $M_2 = 0$ $M_3 = 6$ $M_4 = 53$	$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 1 \end{pmatrix}$
WM4	*All Lines* $M_1 = 0$ $M_2 = 0$ $M_3 = 28$ $M_4 = 56$	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

4 Performance Analysis

4.1 *Comparison with FIR-median Hybrid Filters*

Nieminen *et al* [1] have shown that among all FMH filters 3LH+ has best detail preserving performance and 1LH+ has best noise reduction. Therefore, we take 1LH+, 2LH+ and 3LH+ as the representatives of FMH filters. Our test images are the ring images with different widths, which have been used in [1] where, with these images, the excellent image detail-preserving performance of FMH filters was shown. They are generated using the following formula

$$f(x,y) = \begin{cases} 800, & \text{if } 37 - w \leq \sqrt{(x^2+y^2)} < 37; \\ 200, & \text{otherwise} \end{cases} \tag{9}$$

where $f(x,y)$ is the (x,y)'th pixel of the image and w is the width of the ring. All images are 54×54 pixels. To eliminate the effect of image boundaries we analyze the algorithm only in the 50×50 central area of the image.

First, we give the definition of the root-mean-square error.

Definition 4: Let f be the original image and h the filtered image, then the relative root-mean-square error in each subregion i is defined as

$$v_i = \frac{1}{L}\sqrt{\sum_{(x,y)\in i} (f(x,y) - h(x,y))^2}$$

where the normalizing factor L is given by

$$L = \sqrt{\sum_{(x,y)\in i} f^2(x,y)}.$$

In our comparison process two test images, ring with width 2 and width 1, are used. We assume the rings to represent the desired image details we want to preserve. Each of these test images, which has only two gray levels, can be partitioned into two areas, say *subregion*1 where ring is present and *subregion*2 which represents the whole area of the test image except *subregion*1.

First, let ring images with $w = 2$ and $w = 1$ be filtered by 1LH+, 2LH+, 3LH+, WM1, WM2, WM3 and WM4. The filtered images are shown in Fig. 1(a). It is obvious that the ring with $w = 2$ is preserved by all WM filters and 3LH+. However, 1LH+ and 2LH+ blur the ring. When $w = 1$, the ring is preserved only by WM4. In order to illustrate the difference among them we computed the rms-error in *subregion*1 and *subregion*2, denoted by ε_1 and ε_2 respectively, which are shown in Table 2.

As we have stated earlier, the rings represent our desired image details, thus ε_1 evaluates the detail-preserving ability. And ε_2 represents the effects of filtering operation on non-details area. We also computed ε_0 which takes the whole image as an one-region area and thus it reflects the total effects of filtering operation on the whole image. Next we added white Gaussian noise with zero mean and standard deviation $\sigma = 100$ to the rings and filtered them by these filters. The filtered images are shown in Fig. 1(b). The rms-errors between the original and filtered images are shown in Table 3. From Table 2 and Table 3 we observe that for noisy and noise-free rings, the filter which has the least rms-error is always the WM filters no matter which rms-error ε_0, ε_1 or ε_2 is used. For the noisy rings, WM3 is the best when $w = 1$ and WM2 is better than all FMH filters in the sense of ε_0. However, when $w = 2$, the performance of WM1 becomes the best. WM2 and WM3 are quite similar to WM1 and superior to all FMH filters. One may notice that the performance of WM4 is worse. This is because the detail-preserving ability is so strict that some noise is retained.

Table 2: The rms-errors for noise-free rings

Filter	$w = 1$			$w = 2$		
	ϵ_1	ϵ_2	ϵ_0	ϵ_1	ϵ_2	ϵ_0
Identity	0	0	0	0	0	0
1LH+	0.0661	0	0.0661	0.0253	0	0.0253
2LH+	0.0468	0	0.0468	0.0156	0	0.0156
3LH+	0.0199	0	0.0199	0	0	0
WM1	0.0842	0	0.0842	0	0	0
WM2	0.0515	0	0.0515	0	0	0
WM3	0.0374	0	0.0374	0	0	0
WM4	0	0	0	0	0	0

Table 3: The rms-error for noisy rings

Filter	$w = 1$			$w = 2$		
	ϵ_1	ϵ_2	ϵ_0	ϵ_1	ϵ_2	ϵ_0
Identity	0.0037	0.1824	0.1825	0.0059	0.1488	0.1490
1LH+	0.0626	0.0555	0.0837	0.0264	0.0454	0.0525
2LH+	0.0418	0.0603	0.0734	0.0140	0.0494	0.0513
3LH+	0.0211	0.0900	0.0924	0.0043	0.0733	0.0734
WM1	0.0571	0.0479	0.0745	0.0041	0.0400	0.0402
WM2	0.0399	0.0551	0.0681	0.0042	0.0455	0.0457
WM3	0.0268	0.0595	0.0653	0.0037	0.0483	0.0484
WM4	0.0029	0.0836	0.0837	0.0038	0.0678	0.0679

Rms-errors provide information about both detail preserving and the noise reduction. To measure the noise reduction ability of these filters, we applied them to a constant signal plus white noise. The noise attenuation of these FMH filters and WM filters for Gaussian, uniform and biexponential distributions are plotted in Fig. 2, from which it is observed that the noise reduction of WM1, which has the best noise attenuation among WM1-WM4, is better than that of any FMH filter when the noise is Gaussian or biexponential noise but worse than that of 1LH+ filter when noise is uniformly distributed.

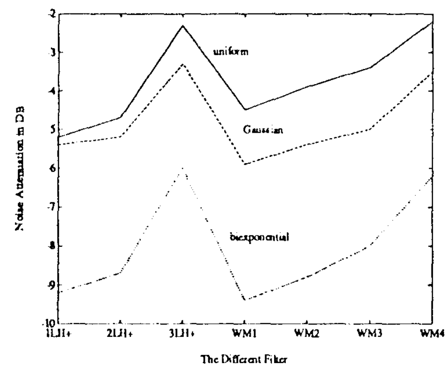


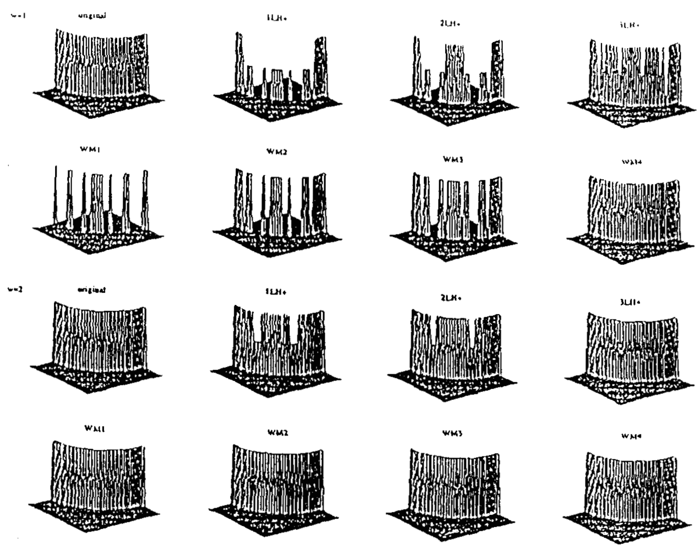
Figure 2: Noise attenuation in constant region

4.2 *Responses under Different signal-to-noise ratios*

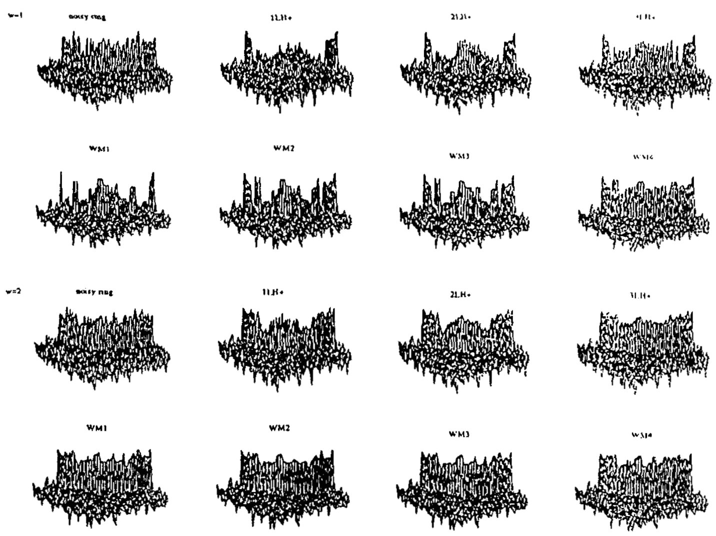
The signal-to-noise ratio (SNR) in an image with two gray levels as was defined by Abdou and Pratt [9]:

$$SNR = \left(\frac{\mu_1 - \mu_2}{\sigma}\right)^2. \qquad (10)$$

Again, we took the ring images with width 1 and 2 as the test images. White Gaussian noise $N(0, 100)$ was added to them. The height of the rings was allowed to change to get noisy images with different SNR: 1, 2, 5, 10, 20, 50 and 100. Then all test images are filtered by WM1, WM2 WM3 and WM4. The rms-errors ε_0's between the original and the filtered images are shown in Fig. 3(a) and (b) for $w = 1$ and $w = 2$, respectively. As mentioned easier, the rms-error



(a)



(b)

Figure 1: Noise free rings (a), and noisy rings (b) with w=1 and w=2 filtered by 1LH+,2LH+,3LH+,WM1,WM2,WM3 and WM4, respectively

reflects both the detail-preserving ability and noise attenuation ability. Thus for $w = 1$, when SNR is low, which means there is a plenty of noise in the test image, the smoothing ability of filter becomes the major factor to reduce ε_0 and thus WM1 is the best. When SNR goes high, the detail-preserving ability plays a more and more important role to reduce ε_0 and thus WM4 becomes the best. However, for $w = 2$, since WM1, WM2, WM3 and WM4 can preserve the ring, their ε_0's behave consistently with increasing of SNR.

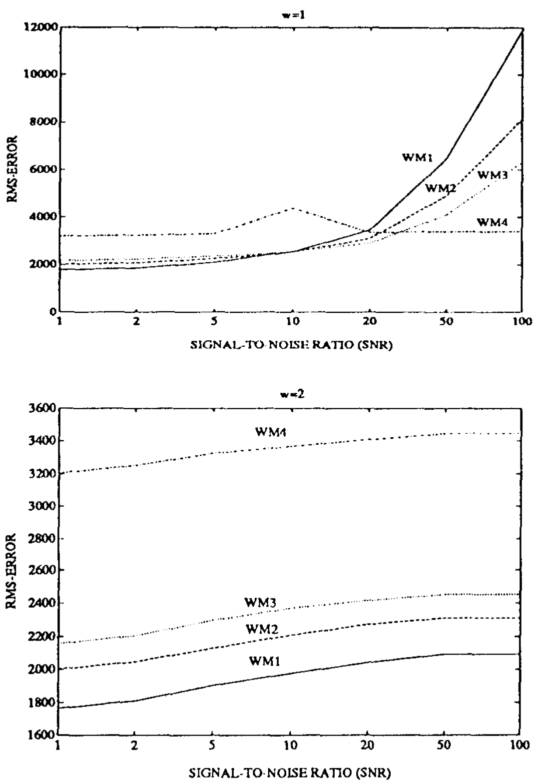


Figure 3: The rms-error for WM1,WM2,WM3 and WM4 under different signal-noise-ratio

5 Conclusions

A design method of detail-preserving WM filters for image processing has been presented. It is shown that optimal WM filters can be found by minimizing the M_i's, a set of parameters which characterize the weights of a WM filter. The performance of these optimal detail-preserving WM filters is compared to that of FIR-median hybrid filters through test images. Simulations show that these detail-preserving WM filters have better performance than FMH filters. Responses under different signal-to-noise ratios are carried out to characterize the performance of these detail-preserving WM filters.

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