# VECTOR MEDIAN-RATIONAL HYBRID FILTERS FOR MULTICHANNEL IMAGE PROCESSING

*Lazhar Khriji*

Electrical Engineering Department, E.N.I.M.
CP 5019 Monastir, Tunisia
E-mail: lazhar.kheriji@enim.rnu.tn

*Moncef Gabbouj*

Signal Processing Laboratory, T.U.T.,
P.O. Box 553 FIN-33 101 Tampere, Finland
E-mail: moncef@cs.tut.fi

## ABSTRACT

In this paper, a new class of nonlinear filters called Vector Median Rational Hybrid Filters (**VMRHF**) for multispectral image processing was introduced and applied to color image filtering problem. These filters are based on Rational Functions (**RF**). The VMRHF is a two-stages filter, which exploits the features of the vector median filter (VM) and those of the vector rational operator (VRF) (The output is the result of vector rational operation taking into account three sub-functions, such as two vector median sub-filters and one center weighted vector median filter (CWVMF)). These filters exhibit desirable properties, such as, edge and details preservation and accurate chromaticity estimation.

The performances of the proposed filter are compared with those of the vector median and the directional-distance filters (DDF).

**Index Terms**: Vector Rational Filters, Vector Median Filters, Vector Median-Rational Hybrid Filters.

## 1. INTRODUCTION

Multichannel image processing is studied in this paper using a vector approach [7] which is more appropriate compared to traditional approaches that have been addressed componentwise operators. This is due to the inherent correlation that exists between the image channels [7]. In vector approaches, each pixel value is considered as an $m-$dimensional vector ($m$ is the number of image channels; in the case of color images, $m = 3$), whose characteristics, i.e., magnitude and direction are examined. The vector's direction signifies their chromaticity, while their magnitude is a measure of their brightness. A number of vector processing filters usually involve the minimization of an appropriate error criteria [1], [9]. One class of filters considers the distance in the vector space between the image vectors; typical representative of this class is the "vector median filter" (VMF) [1]. A second class of filters operate by considering the vectors' direction, and hence the name "vector directional filters" (VDF) [9].

VMF's are derived as MLE estimates from exponential distribution [1], while VDF's are spherical estimators, when the underlying distribution is a spherical one [9]. The former VMF's perform accurately when the noise follows a long-tailed distribution (e.g. exponential or impulsive); moreover, any outliers in the image data are easily detected and eliminated by VMF's. A third class of filters operate using rational functions in their input/output relation, and hence the name "vector rational filters" (VRF's) [3]. There are several advantage to the use of this function. Similarly to a polynomial function, a rational function is a universal approximator (it can approximate any continuous function arbitrarily well); however, it can achieve a desired level of accuracy with a lower complexity, and possesses better extrapolation capabilities. Moreover, it has been demonstrated that a linear adaptive algorithm can be devised for determining the parameters of this structure [6].

In this paper, a novel filter structure is introduced, the vector median-rational hybrid filter's (VMRHF's), which constitute a natural extension of the nonlinear rational type hybrid filters called median-rational hybrid filter's (MRHF's) recently introduced for 1-D and 2-D signal processing [4], [5], based on rational functions. The VMRHF is formed by three sub-filters (in which two vector median filters and one center weighted vector median filter) and one vector rational operation. VMRHF are very useful in color (and generally multichannel) image processing, since they inherit the properties of their ancestors. They constitute very accurate estimators in long- and short-tailed noise distributions and, at the same time, they preserve the chromaticity of the image vectors. Moreover, they act in small window and few number of operations, resulting in simple and fast filter structures.

## 2. RATIONAL AND VECTOR RATIONAL FUNCTIONS

A rational functions is the ratio of two polynomials. To be used as a filter, it can be expressed as:

$$y = \frac{a_0 + \sum_{i=1}^{m} a_{1i}x_j + \sum_{i=1}^{m}\sum_{j=1}^{m} a_{2ij}x_ix_j + \dots}{b_0 + \sum_{i=1}^{m} b_{1i}x_i + \sum_{i=1}^{m}\sum_{j=1}^{m} b_{2ij}x_ix_j + \dots}, \quad (1)$$

0-7803-5471-0/99/$10.00©1999 IEEE

where $x_1, x_2, ..., x_m$ are the scalar inputs to the filter and $y$ is the filter output, $a_0, b_0, a_{ij}$ and $b_{ij}$ are filter parameters.

Straight forward application of the rational functions to multichannel image processing would be based on processing the image channels separately. This however, fails to utilize the inherent correlation that is usually present in multichannel images. Consequently, vector processing of multichannel images is desirable [7]. The generalization of the scalar rational filter definition to vector and scalar signals alike is given by the following definition.

**Definition 2.1** *Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m$ be the $m$ input vectors to the filter, where $\mathbf{x_i} = \left[x_i^1, x_i^2, \cdots, x_i^l\right]^T$ and $x_i^k \in \{0, 1, \cdots, M\}$, $M$ is an integer. The VRF output is given by*

$$\begin{aligned} VRF &= RF\left[\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m\right] \qquad (2) \\ &= \frac{P(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)}{Q(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)} = \left[rf_1, rf_2, \ldots, rf_l\right]^T \end{aligned}$$

*where $P$ is a vector-valued polynomial and $Q$ is a scalar polynomial. Both are functions of the input vectors. The $i^{th}$ component of the VRF output is written as*

$$rf_i = \left\lfloor \frac{P_i(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)}{Q(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)} \right\rfloor \in \{0, 1, \ldots, M\} \qquad (3)$$

*where*

$$\begin{aligned} P_i(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m) &= a_0 + \sum_{k=1}^{m} a_k x_k^i \qquad (4) \\ &+ \sum_{k_1=1}^{m} \sum_{k_2=1}^{m} a_{k_1 k_2} x_{k_1}^i x_{k_2}^i + \cdots \end{aligned}$$

*and*

$$Q(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m) = b_0 + \sum_{j=1}^{m} \sum_{k=1}^{m} b_{jk} ||\mathbf{x}_j - \mathbf{x}_k||_p \qquad (5)$$

*$||.||_p$ is the $l_p$-norm, and $\lfloor \alpha \rfloor$ = integer part of $\alpha$, $\alpha \in \mathcal{R}_+$. $b_0 > 0$, $b_{ij}$ are constant, and $a_{i_1, i_2, \ldots, i_n}$ is a function of the input vectors:*

$$a_{i_1, i_2, \ldots, i_n} = f(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m) \qquad (6)$$

*When the vector dimension is 1, the VRF reduces to a special case of the scalar RF [3].*

## 3. VECTOR MEDIAN-RATIONAL HYBRID FILTERS (VMRHF)

When extending the median-rational hybrid operation to vector-valued signals, we place some requirements for the resulting vector median-rational hybrid operation:

- The operation should have properties similar to those of the scalar case.
- Good robust data smoothing ability for different i.i.d. noise distributions (Gaussian, impulsive, mixed Gaussian-impulsive), while retaining sharp edges in the signal.
- It reduces to the scalar filter if the vector dimension is 1.

**Vector median-rational hybrid filters:**

Let $\underline{\mathbf{f}}(x) : Z^l \to Z^m$, represent a multichannel signal and let $W \in Z^l$ be a window of finite size $n$ (filter length). $l$ represents the signal dimensions and $m$ represents the number of signal channels. The pixels in $W$ will be denoted as $x_i, i = 1, 2, \ldots, n$ and $\underline{\mathbf{f}}(x_i)$ will be denoted as $\underline{\mathbf{f}}_i$. $\underline{\mathbf{f}}_i$ are $m$-dimensional ($m \geq 2$) vectors in the vector space defined by the $m$ signal channels. the VMRHF is introduced as follows:

**Definition 3.1** *The output vector $\underline{y}(\underline{\mathbf{f}}_i)$ of the VMRHF, is the result of a vector rational function taking into account three input sub-functions which form an input functions set $\{\underline{\Phi}_1, \underline{\Phi}_2, \underline{\Phi}_3\}$, where the "central one" ($\underline{\Phi}_2$) is fixed as a center weighted vector median sub-filter*

$$\underline{y}(\underline{\mathbf{f}}_i) = \underline{\Phi}_2(\underline{\mathbf{f}}_i) + \frac{\sum_{j=1}^{3} \alpha_j \underline{\Phi}_j(\underline{\mathbf{f}}_i)}{h + k||\underline{\Phi}_1(\underline{\mathbf{f}}_i) - \underline{\Phi}_3(\underline{\mathbf{f}}_i)||_2} \qquad (7)$$

*where, $||.||_2$ is an $L_2$-vector norm, $\alpha = [\alpha_1, \alpha_2, \alpha_3]$ characterizes the constant vector coefficient of the input sub-functions. In this approach, we have chosen a very simple prototype filter coefficients which satisfies the condition: $\sum_{i=1}^{3} \alpha_i = 0$. In our study, $\alpha = [1, -2, 1]^T$. $h$ and $k$ are some positive constants. The parameter $k$ is used to control the amount of the nonlinear effect.*

The sub-filters $\underline{\Phi}_1$ and $\underline{\Phi}_3$ are chosen so that an acceptable compromise between noise reduction, edge and chromaticity preservation. It is easy to observe that this VMRHF differs from a linear low-pass filter mainly for the scaling, which is introduced on the $\underline{\Phi}_1$ and $\underline{\Phi}_3$ terms. Indeed, such terms are divided by a factor proportional to the output of an edge-sensing term characterized by $l_2-$vector norm of the vector difference between the two vectors $\underline{\Phi}_1$ and $\underline{\Phi}_3$. The weight of the vector median-operation output term is accordingly modified, in order to keep the gain constant. The behavior of the proposed VMRHF structure for different positive values of parameter $k$ is the following:

**1:** $k \simeq 0$, the form of the filter is given as a linear low-pass combination of the three nonlinear sub-functions:

$$\underline{y}(\underline{\mathbf{f}}_i) = c_1 \underline{\Phi}_1(\underline{\mathbf{f}}_i) + c_2 \underline{\Phi}_2(\underline{\mathbf{f}}_i) + c_3 \underline{\Phi}_3(\underline{\mathbf{f}}_i). \qquad (8)$$

where, the coefficients $c_1$, $c_2$, and $c_3$ are some constants.

**2:** $k \to \infty$, the output of the filter is identical to the central sub-filter output and the vector rational function has no effect:

$$\underline{y}(\underline{\mathbf{f}}_i) = \underline{\Phi}_2(\underline{\mathbf{f}}_i). \quad (9)$$

**3:** For intermediate values of $k$, the $||\underline{\Phi}_1(\underline{\mathbf{f}}_i) - \underline{\Phi}_3(\underline{\mathbf{f}}_i)||_2$ term perceives the presence of a detail and accordingly reduces the smoothing effect of the operator.

Therefore, the VMRHF operates as a linear lowpass filter between three nonlinear suboperators, the coefficients of which are modulated by the edge-sensitive component. The proposed structure of the VMRHF is shown by Fig. 1 using two bidirectional vector median sub-filters.

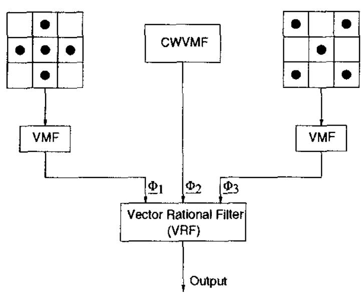



Figure 1: Structure of VMRHF using bidirectional sub-filters.

## 4. EXPERIMENTAL RESULTS

The vector median-rational hybrid filters have been evaluated, and their performance has been compared against the performance of the widely known multidimensional nonlinear filters; VMF and DDF, using RGB colors images as test multidimensional data.

The noise attenuation properties of the different filters are examined by utilizing the color image roze (see fig. 2). The test image has been contaminated using various noise source models in order to assess the performance of the filters under different scenarios:

**1:** Gaussian noise implies corruption by zero mean additive noise with standard deviation $\sigma$, $\mathcal{N}(0, \sigma^2)$ .

**2:** Impulsive noise: each image channel is corrupted independently using salt and pepper noise. we assume that both salt and pepper are equally likely to occur.

**3:** Mixed Gaussian-impulsive noise: the impulsive noise is fix (salt and pepper 2% in each image channel), the Gaussian noise $\mathcal{N}(0, \sigma^2)$.

The original image, as well as its noisy versions, are represented in the RGB color space. This color coordinate system is considered to objective, since it is based on the physical measurements of the color attributes. The filters operate on the images in the RGB color space.

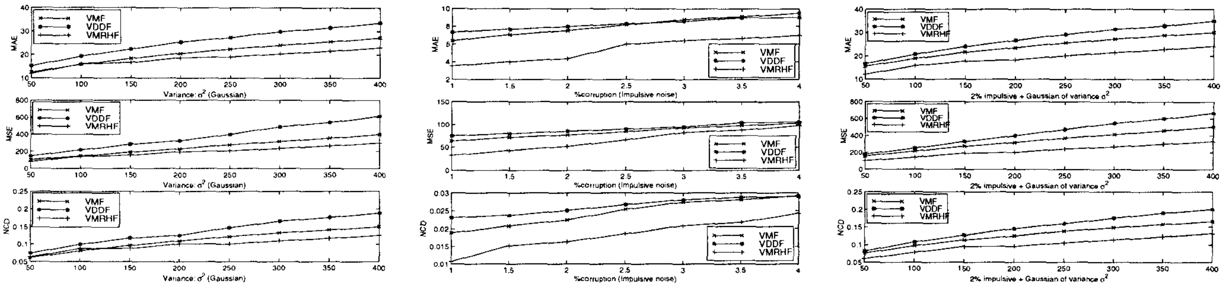
A number of different objective measures can be utilized for quantitative comparison of the performance of the different filters. All of them provide some measure of closeness between two digital images by exploiting the differences in the statistical distributions of the pixel values [2]. The most widely used measures are the mean absolute error ($MAE$), the mean square error ($MSE$), and the normalized color difference ($NCD$). The latter measure is used to quantify the perceptual error between images in the perceptually uniform $L^*a^*b^*$ color space which is known as a space where equal color differences result in equal distances [8].

The results obtained are shown in the form of plots in Fig.3 for the three noise models: Gaussian, impulsive, and Gaussian mixed with impulsive, respectively. As can be verified from the plots, the performance of the new VMRHF is superior to the performance of VMF and DDF. Moreover, consistent results have been obtained when using a variety of other color images and the same evaluation procedure.

The filtered images are presented in Fig.4 for visual and qualitative comparison, since in many cases they are the best qualitative measure of the performance of image processing techniques. Figs. 4(a)-4(d) are the corrupted image by mixed noise (impulsive 2% in each channel, Gaussian $\mathcal{N}(0, 50)$), the filtered images using DDF, VMF, and VMRHF respectively. All the filters considered operate using a square 3x3 processing window. A comparison of the images clearly favors our newly VMRHF over their counterparts (VMF and DDF). The proposed VMRHF can effectively remove impulses, smooth out nominal noise and keep edges, details and color uniformity unchanged. Considering the number of computations required for the implementation of the VMRHF, it should be noted that it is comparable of those of VMF. The vector rational operation does not introduces significant additional computational cost, (a small look-up table for the denominator, one multiplication, three additions and one division per output sample).

## 5. CONCLUSION

New class of nonlinear vector rational type hybrid filters for multidimensional image processing was introduced in this paper. The vector median-rational hybrid filter is a two-stages filter, which exploits the features of the vector median filter and those of the vector rational operator. It acts as a vector rational operation of three sub-filters in which the central one is a central weighted vector median filter. Simulation results and subjective evaluation of the filtered images indicate that the VMRHFs outperform all other filters under consideration. Moreover, as it can be seen from the processed images, the VMRHF preserve the chromaticity component.




(a) (b) (c)

Figure 3: Comparative results from the image in Fig.2 contaminated by: (a) Gaussian, (b) Impulsive, and (c) mixed noise (salt and pepper 2% in each component).

(a) (b) (c)

Figure 4: Results on the roze image [Fig.2]. Images (a), (b) and (c) are the processed images by the DDF, VMF and VMRHF, respectively.

## 6. REFERENCES


[1] J. Astola, P. Haavisto and Y. Neuvo, "Vector Median Filter", *Proceedings of the IEEE,* vol.78, pp.678-689, April 1990.

[2] A.M. Eskicioglo, P.S. Fisher, and S. Chen, "Image quality measures and their performance",*IEEE Trans. on Communication*, vol. 43, pp. 2959-2965, 1995.

[3] L. Khriji, F.A. Cheikh and M. Gabbouj, "Multistage vector rational interpolation for color images", *In Proc. of the Second IMACS-IEEE International Multiconference on Computational Engineering in System Application: CESA'98*, Hammamet, Tunisia, April 1-4, 1998.

[4] L. Khriji and M. Gabbouj, "Median-Rational Hybrid Filters for image restoration", *IEE Electronics Letters*, vol.34, no.10, pp.977-979, May 1998.

[5] L. Khriji and M. Gabbouj, "Median-Rational Hybrid Filters",*Intern. Conf. on Image Processing ICIP'98*, Chicago, Illinois, USA. October 4-7, 1998.

[6] H. Leung and S. Haykin, "Detection and estimation using an adaptive rational function filter",*IEEE Trans. on Signal Processing*, Vol.42, no.12, pp.3366-3376, December 1994.

[7] R. Machuca, K. Phillips, "Applications of vector fields to image processing",*IEEE Trans. Pattern Anal. Mach. Intell.*, vol. PAMI-5, pp. 316-329, May 1983.

[8] W.K. Pratt, "Digital Image Processing",*John Wiley and Sons,* New York, NY, 1991.

[9] P.E. Trahanias, D. Karakos and A.N. Venetsanopoulos, "Directional Processing of Color Images: Theory and Experimental Results",*IEEE Trans. on Image Processing*, vol. 5, no. 6, pp. 868-880, June 1996

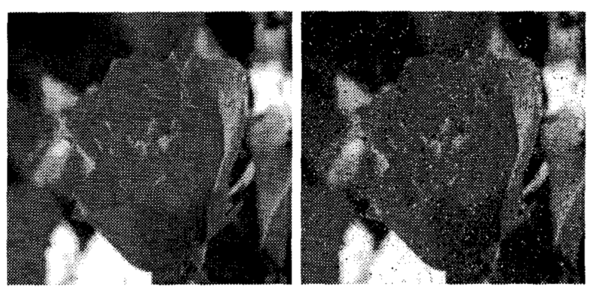



Figure 2: (left) Test color roze image, (right) Contaminated image by mixed noise (impulsive 2 % in each channel and Gaussian $\mathcal{N}(0, 50)$)