Pattern Recognition Letters 23 (2002) 215–220

Pattern Recognition Letters

www.elsevier.com/locate/patrec

Wavelet-based corner detection technique using optimal scale

Azhar Quddus *, Moncef Gabbouj

Signal Processing Laboratory, Tampere University of Technology (TUT), P.O. Box 553, FIN-33101, Tampere, Finland

Received 24 October 2000; received in revised form 3 May 2001

Abstract

In this paper we present a novel technique for wavelet-based corner detection using singular value decomposition (SVD). Here SVD facilitates the selection of global natural scale in discrete wavelet transform. We define natural scale as the level associated with most prominent (dominant) eigenvalue. Eigenvector corresponding to dominant eigenvalue is considered as the optimal scale. The corners are detected at the locations corresponding to modulus maxima. Results indicate the suitability of the approach. Comparison with a recently proposed technique is also provided. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Shape analysis; Corner detection; Wavelet decomposition; Dyadic decomposition; Feature extraction

1. Introduction

Corners in digital images give important clues for shape representation and analysis (Attneave, 1954). Wavelet analysis finds many diverse applications including communications, computer graphics, computer vision, image processing and geophysics. It is already established that the wavelet orthonormal bases provide a useful multiresolutional signal representation and a tool for signal analysis (Mallat, 1989). Recently, there have been work reported for content-based image retrieval systems using corners (high curvature points) to extract features (Quddus et al., 1999, 2000). These techniques have the advantage of being computationally fast over large image databases.

Research has been reported on corner detection using wavelets (Lee et al., 1995; Quddus and Fahmy, 1999). Existing approaches do not analyze wavelet decomposition at the natural scale. It is well known that at lower level of the decomposition, more details are available at the expense of higher noise. On the other hand, at higher scales, more details are missed while noise is reduced considerably. However, there is research reported on selection of natural scale for different problems (Witkin, 1984; Rosin and Venkatesh, 1993; Rosin, 1994). The approaches discussed in these reports are suitable for scale-space filtering. These approaches are not suitable for dyadic discrete wavelet domain. These techniques require complete decomposition before the natural scales could be detected. Hence, they do not provide any

* Corresponding author. Present address: Veracel Inc., 59 Adelaide Street East, Toronto, Canada M5C 1K6. Tel.: +416-361-6232; fax: +416-361-0427.

E-mail address: aquddus@morphx.com (A. Quddus).

PII: S0167-8655(01)00090-3

information as to where the decomposition should be stopped. The approach discussed by Fdez-Valdivia et al. (1998) is based on 2D Gabor filtering.

We have recently proposed an elegant technique for the selection of natural scales in discrete wavelet domain (Quddus and Gabbouj, 2000). Here we extend this approach to detect corners. This technique adaptively detects optimal scale while decomposition is being carried out. The technique is based on singular value decomposition (SVD).

2. Extraction of wavelet at natural scale

The preprocessing steps (silhouette extraction, boundary tracking and tangent computation) are the same as those in (Lee et al., 1995). This preprocessing gives the orientation profile (orientation of the tangent along the boundary contour) from the input image.

Let the orientation profile be rewritten as $D = (S_1 f(n))_{n \in Z}$. Here we keep notations same as Mallat and Zhong (1992). Let

$$W_{2^j}^d f = (W_{2^j} f(n+\omega))_{n \in Z} \tag{1}$$

and

$$S_{2^j}^d f = (S_{2^j} f(n+\omega))_{n \in Z}, \tag{2}$$

where ω is the sampling shift that depends only on the wavelet $\varphi(x)$. For any coarse scale 2^J, the sequence of discrete signals,

$$\left\{ S_{2^J}^d, \left(W_{2^j}^d f \right)_{1 \leqslant j \leqslant J} \right\} \tag{3}$$

is called the *discrete dyadic wavelet transform* of $D = (S_1 f(n))_{n \in Z}$. Here $S_{2^J}^d$ is *the last approximation* and the set of sequence $(W_{2^j}^d f)_{1 \leqslant j \leqslant J}$ is *wavelet coefficients* or *details* at levels $2^1 \leqslant 2^j \leqslant 2^J$.

It has been shown that the quadratic wavelets (Mallat and Zhong, 1992) perform very well for corner detection application (Quddus and Fahmy, 1999).

Let *wavelet details* $(W_{2^j}^d f)_{1 \leqslant j \leqslant J}$ be represented in matrix P with each row representing the details at levels 2^j starting from level 2^1. Hence,

$$P = \begin{bmatrix} W_{2^1}^d f \\ W_{2^2}^d f \\ \vdots \\ W_{2^J}^d f \end{bmatrix}. \tag{4}$$

The matrix P is not square hence, its eigenvalues can be computed using SVD, where

$$\mathrm{SVD}\{P\} = T \cdot S \cdot E'. \tag{5}$$

Here $(\cdot)$ indicates matrix multiplication and $(')$ indicates transpose. If length of sequence D is M, where $J < M$, then matrices T, S and E have the forms:

$$T = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1M} \\ T_{21} & T_{22} & \cdots & T_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ T_{J1} & T_{J2} & \cdots & T_{JM} \end{bmatrix}, \tag{6}$$

$$S = \begin{bmatrix} S_1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & S_2 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & S_j & 0 & 0 & \cdot & 0 \\ 0 & \cdot & 0 & S_J & 0 & \cdot & 0 \end{bmatrix}, \tag{7}$$

$$E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1M} \\ E_{21} & E_{22} & \cdots & E_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ E_{M1} & E_{M2} & \cdots & E_{MM} \end{bmatrix} = \begin{bmatrix} E_1^* \\ E_2^* \\ \vdots \\ E_M^* \end{bmatrix}, \tag{8}$$

where $S_1 \geqslant S_2 \geqslant \cdots \geqslant S_j \geqslant \cdots \geqslant S_J$. Matrix E contains eigenvectors.

Now Eq. (6) can be rewritten as

$$\begin{aligned} \left(W_{2^j}^d f \right)_{1jJ} &= \sum_{m=1}^{M} T_{jm} \left(S_m E_m^* \right) \\ &= T_{j1} \left(S_1 E_1^* \right) + \sum_{m=2}^{M} \left(T_{jm} \left(S_m E_m^* \right) \right). \end{aligned} \tag{9}$$

If $S_1 \gg \{S_2, S_3, \ldots S_j, \ldots, S_J\}$, then

$$\left(W_{2^j}^d f \right)_{1 \leqslant j \leqslant J} \approx T_{j1} \left(S_1 E_1^* \right) \tag{10}$$

and hence S_1 detects dominant mode in P. In other words, S_1 detects most dominant behavior among wavelet details.

The *level* corresponding to the *dominant mode* of the $(W_{2^j}^d f)_{1 \leqslant j \leqslant J}$ is defined as *natural scale*. The detection of dominant scale using this approach requires wavelet decomposition be computed at all possible scales. Hence, we start with only first two levels (i.e. 2^1 and 2^2) and compute SVD adaptively while adding higher levels. Let, for any level 2^k, X_{2^k} be the matrix extracted by selecting first k rows of P. Let $\{S_1^X \; S_2^X \; \cdots \; S_k^X\}$ be the eigenvalues of X_{2^k}. We define a quantity *dominant mode difference*, D^{Xk}, as

$$D^{Xk} = S_1^X - \sum_{l=2}^{k} S_l^X. \tag{11}$$

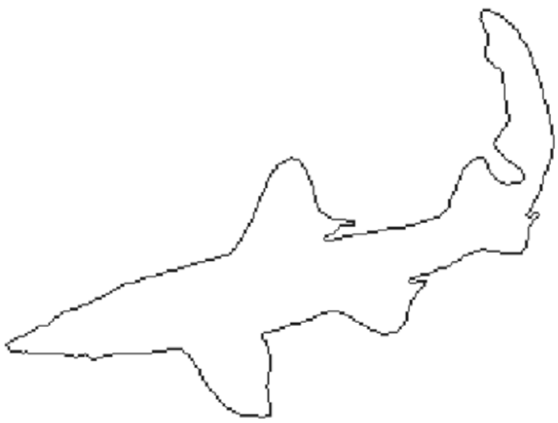

Fig. 1. Input image.

Now finding natural scale converges to find k such that D^{Xk} is maximum. We now provide the algorithm to detect natural scale.

Algorithm.

1. Compute two level wavelet details (i.e., X_{2^2}) and set $j = 2$.
2. Compute D^{Xj}.
3. *while* $(D^{X(j+1)} - D^{Xj}) \geqslant 0$
   $j = j + 1$
   compute X_{2^j} and D^{Xj}
   endwhile
4. Select natural scale as $L = 2^j$.

Hence, X_{2^L} gives dominant mode behavior in scales $2^1 \leqslant 2^j \leqslant 2^L$, where

$$X_{2^L} = \left(W_{2^j}^d f\right)_{1 \leqslant j \leqslant L} = T_{j1}\left(S_1 E_1^*\right). \tag{12}$$

A good estimate of the wavelet decomposition at the natural scale $W_{2^*}^d f$ can be obtained as

$$W_{2^*}^d f = \frac{1}{L} \sum_{j=1}^{L} W_{2^j}^d f. \tag{13}$$

We define *normalized modulus maxima* of $W_{2^*}^d f$ as

$$\left(W_{2^*}^d f\right)_{\text{norm}} = \frac{|W_{2^*}^d f|}{\max\{|W_{2^*}^d f|\}}, \tag{14}$$

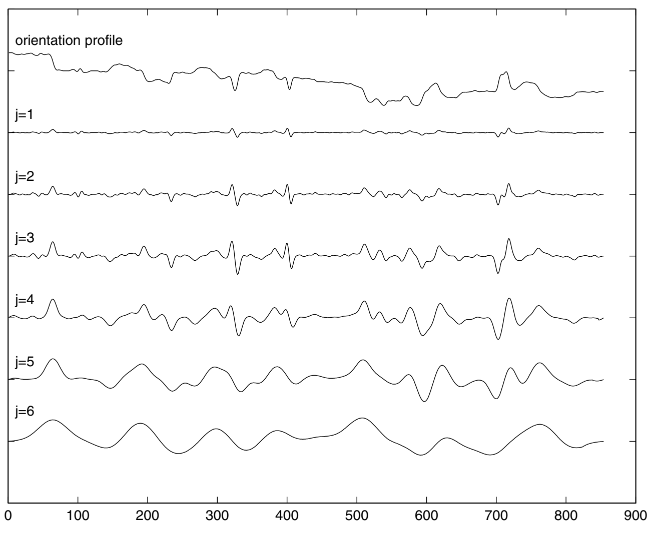

Fig. 2. Orientation profile and wavelet decomposition.

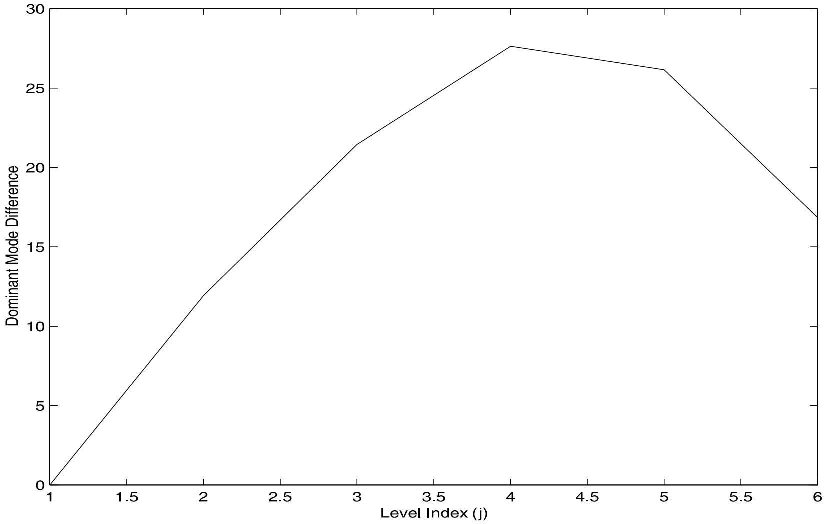

Fig. 3. Dominant mode difference for Fig. 2.

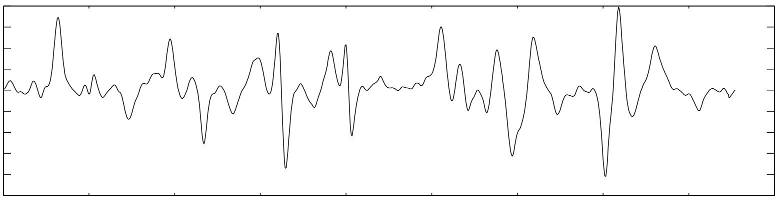

Fig. 4. Estimated wavelet decomposition at natural scale.

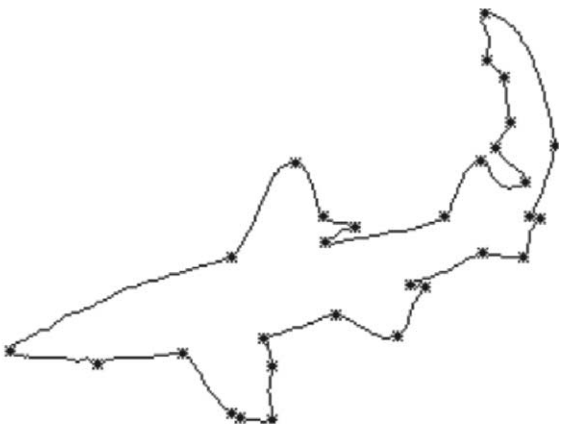

Fig. 5. Detected corners for Fig. 1.

where $|\cdot|$ shows modulus maxima detected using the approach in (Mallat and Zhong, 1992).

Now corners are detected as the locations where $(W^d_{2^*}f)_{\mathrm{norm}}$ is greater than a certain threshold ε.

3. Results and discussions

Fig. 1 shows the input image. Orientation profile and its wavelet decomposition is shown in Fig. 2. For the sake of illustration decomposition is shown more levels than the required. The dominant mode difference is shown in Fig. 3. Here we observe that optimal natural scale is 2^4. Fig. 4 shows the estimated wavelet decomposition at natural scale. Fig. 5 shows the detected corners marked with '$*$'. Here we used $\varepsilon = 0.15$. Similar corner detection results are shown in Figs. 6–8 with $L = 4$, 5 and 3, respectively.

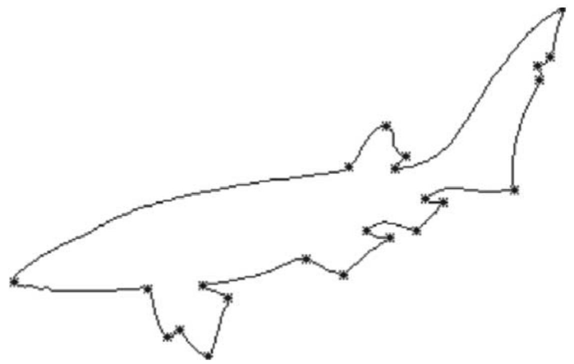

Fig. 6. Corner detection result with $L = 4$.

Fig. 7. Corner detection result with $L = 5$.

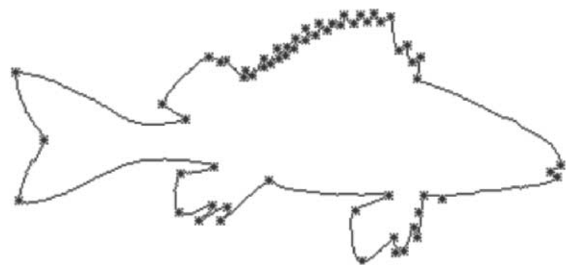

Fig. 8. Corner detection result with $L = 3$.

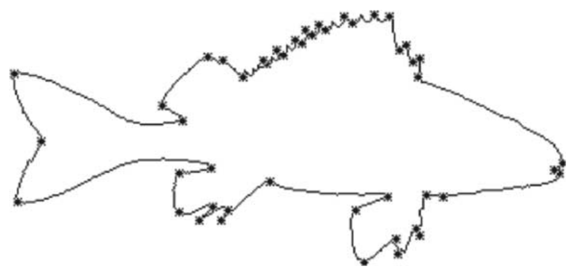

Fig. 9. Corner detection result using method of Quddus and Fahmy (1999).

Fig. 9 shows the result obtained by the technique proposed in (Quddus and Fahmy, 1999). In that work, the analysis always starts with the level 2^4 and hence the results are not always the best. However, in Fig. 8 we obtain a better result by doing the analysis at natural scale. For this image natural scale was found to be at level 2^3.

4. Conclusions

We have presented a novel approach for corner detection where selection of wavelet decomposition at natural scale is done using SVD. We define the natural scale as decomposition corresponding to dominant mode (largest eigenvalue). Algorithm is given for the selection of natural wavelet scale under discrete wavelet domain. Corners are extracted at modulus maxima and results have been provided. We have also shown the comparison of this approach with a recently proposed technique.

References

Attneave, F., 1954. Some informational aspects of visual perception. Psychol. Rev. 61, 183–193.

Fdez-Valdivia, J., Garcia, J.A., Martinez-Baena, J., 1998. The selection of natural scales in 2D images using adaptive Gabor filtering. IEEE Trans. Pattern Anal. Mach. Intell. PAMI-20 (5), 458–469.

Lee, J.S., Sun, Y.N., Chen, C.H., 1995. Multiscale corner detection by wavelet transform. IEEE Trans. Image Process. 4, 100–104.

Mallat, S.G., 1989. A theory for multiresolution signal decomposition: the wavelet representation. IEEE Trans. Pattern Anal. Mach. Intell. PAMI-11, 674–693.

Mallat, S., Zhong, S., 1992. Characterization of signals from multiscale edges. IEEE Trans. Pattern Anal. Mach. Intell. PAMI-14 (7), 710–732.

Quddus, A., Fahmy, M.M., 1999. Fast wavelet-based corner detection technique. IEE Electron. Lett. 35, 287–288.

Quddus, A., Cheikh, F.A., Gabbouj, M., 1999. Wavelet-based multi-level object retrieval in contour images. In: Very Low Bit Rate Coding (VLBV'99), Kobe, Japan.

Quddus, A., Gabbouj, M., 2000. Selection of natural scale in discrete wavelet domain using eigenvalues. In: Xth Eur. Signal Process. Conf., Tampere, Finland, 2000.

Quddus, A., Cheikh, F.A., Gabbouj, M., 2000. Content-based object retrieval using maximum curvature points in contour images. In: SPIE – Electronic Imaging Conf., San Jose, CA, USA.

Rosin, P.L., 1994. Determining local natural scales of curves. In: Proc. 12th IAPR Internat. Conf. on Pattern Recognition, Vol. 1, pp. 185–189.

Rosin, P., Venkatesh, S., 1993. Extracting natural scales using Fourier descriptors. Pattern Recognition 26, 1383–1393.

Witkin, A.P., 1984. Scale-space filtering: a new approach to multi-scale description. Proc. Image Understanding, 79–95.