1 Math problem 1p

Consider a 3-state left-to-right discrete output-symbol HMM with parameters shown below. What is the probability of the output sequence \((O) = AABCBBBC\) ?

\[
\begin{align*}
    a_{11} &= 0.9 \quad a_{12} = 0.1 \quad a_{13} = 0.0 \\
    a_{21} &= 0.0 \quad a_{22} = 0.9 \quad a_{23} = 0.1 \\
    a_{31} &= 0.0 \quad a_{32} = 0.0 \quad a_{33} = 1.0 \\
    b_1(A) &= 0.8 \quad b_1(B) = 0.1 \quad b_3(C) = 0.1 \\
    b_2(A) &= 0.2 \quad b_2(B) = 0.2 \quad b_3(C) = 0.6 \\
    b_3(A) &= 0.3 \quad b_3(B) = 0.1 \quad b_3(C) = 0.6
\end{align*}
\]

2 Matlab 2p: Viterbi algorithm

Implement the Viterbi algorithm. For the HMM and observation sequence in Problem 1, what is the most likely state sequence \(q\) and the probability \(P(O|q)\)?

3 Matlab 2p: Forward/backward algorithm

Implement the forward and backward algorithms for discrete output HMMs, according to the interface [http://www.cs.tut.fi/~sgn24006/exercises/forward_backward.m](http://www.cs.tut.fi/~sgn24006/exercises/forward_backward.m)

Test that with the HMM with parameters A and B shown below and \((O) = 1, 1, 2, 2, 2, 3, 2, 1\). If you get the value 0.00010914 your implementation is correct (or the reference implementation is wrong).

You can actually calculate the total probability by \(\sum_t \alpha_t(i)\beta_t(i)\) for any \(t\) to verify this.
\[ A = \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.15 & 0.80 & 0.05 \\ 0.10 & 0.20 & 0.70 \end{bmatrix} \]

\[ B = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix} \]