Phase Noise Mitigation in OFDMA Uplink

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Abstract—This paper addresses the problem of the phase noise induced spectral spread in an OFDMA uplink scenario. Accompanied with power differences between the users at the receiver in a multiuser case, the phase noise induced spread causes performance degradation for users with low signal powers. This paper proposes a receiver-side digital signal processing methods to estimate and mitigate the spread caused by the transmitter-side phase noise. The introduced methods show a significant gain when compared with other conventional phase noise estimation and mitigation methods that are not designed especially for the multiuser scenario.

Keywords—OFDMA; Phase Noise; Uplink; Mitigation.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique used in the digital transmission of data. Its robustness against frequency selective channels and ease of implementation using the Fast Fourier Transform (FFT) algorithm are a few of many advantages that favor its selection over other modulation techniques to cope with the increasing demand on high data rate communications. However, OFDM is highly sensitive to frequency errors. One of the sources of frequency errors is the oscillator phase noise (PN), and many studies in the literature have attempted to mitigate this major source of performance degradation in OFDM [1], [2], [3], [4], [5] and [6]. Basic PN mitigation techniques involve the estimation of the Common Phase Error (CPE) [1], while other more advanced mitigation techniques involve the estimation of Inter-Carrier Interference (ICI) as well [2], [3], [4], [5] and [6].

In Orthogonal Frequency Division Multiple Access (OFDMA), in-band ICI is not the only source of interference that should be considered [7]. The multiplexing of several users in an OFDMA uplink scenario introduces out-of-band interference from one user on another in the OFDMA symbol. This interference is induced by the spectral spread of the energy of each user’s subcarriers on the top of other users’ subcarriers. The spread is more severe when there is an unequal power level for each user due to different path loss effects for each user in an uplink scenario [7]. Ideally, a power control algorithm is applied on the users in a network, and all users should have similar power levels at the receiver, while this is not the case in practice.

In [6], a PN estimation and mitigation technique was proposed in the case where the adjacent channel interference due to receiver PN is present, although the paper did not consider transmitter PN, which is more significant in OFDMA. In this paper, an OFDMA uplink scenario is considered where each OFDMA user suffers from the transmitter PN effect and causes interference. This transmitter PN induced interference, which is manifested by a spectral spread, is then estimated and compensated at the receiver to boost the Bit-Error-Rate (BER) performance of a user that suffers from a PN induced spread of other users’ signals on top of its own, which can be referred to as Inter-User Interference (IUI). The simulations in this paper consider varying the power levels and the PN 3-dB bandwidth of the users as well as the conventional BER vs. SNR performance with and without the mitigation of IUI.

II. PHASE NOISE AND OFDMA-LINK MODELLING

This section explains the PN model used in this paper along with the OFDMA uplink model.

A. Phase Noise Modeling

In this paper, the PN process is modeled using the Free-Running Oscillator (FRO) model. The FRO model is a mathematical realization that models the PN realization as a random process called Brownian motion [8]. The FRO model is easy to implement in a discrete-time simulation and provides a similar effect on OFDM signals that more practical oscillator models cause [6]. It can be written as

\[
\phi_n = \sqrt{2B}(nT_j)
\]

where \(\phi_n\) is the discrete time PN, \(B(nT_j)\) is the discrete-time varying standard Brownian motion [8], \(c\) represents the inverse of the relative oscillator phase noise which is referred to as the diffusion rate of the Brownian motion process and \(T_j\) is the sampling time interval. Statistically, the difference between two samples of a Brownian motion process follows a normal distribution which can be characterized with the property

\[
B(nT_j) - B((n + 1)T_j) \sim N(0, cT_j).
\]

Equation (2) means that a sampled FRO process can be generated by using a cumulative sum of the realizations of normal distributed samples with a zero-mean and variance \(cT_j\). The spectrum of a standard Brownian motion has a Lorentzian shape [8]; therefore, the 3-dB bandwidth of the PN \(\beta\) can be written as

\[
\beta = \frac{c}{4\pi},
\]

where \(\beta\) is in Hertz. The 3-dB bandwidth is the parameter used to control the intensity of the PN effect in an OFDM symbol.

B. OFDMA Uplink Modeling

The OFDMA symbol is transmitted by applying the Inverse Fast Fourier Transform (IFFT) algorithm on the modulated data subcarriers. The total number of subcarriers \(M\) of a certain user \(q\) depends on the number of users in the OFDMA symbol, while the rest of the subcarriers that are reserved for other users are null, and the total size of the OFDMA symbol is \(N\). The \(n\)th sample of the \(n\)th OFDMA symbol of the \(q\)th OFDMA user can be written as

\[
x_{n,q}(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{M-1} x_{k,q}(m)e^{j2\pi kn/N},
\]

where \(x_{n,q}(m)\) denotes the transmitted samples and \(x_{k,q}(m)\) are the modulated subcarriers where \(k \in [0, N-1]\). \(x_{k,q}(m)\) is always constructed so that the desired subcarrier mapping is achieved (contiguous or distributed). At the receiver, the signals of all OFDMA...
users, are summed up together to finally have the AWGN added to the whole received OFDMA symbol. Notice that the signals of all the users experience different transmitter PN processes and multipath channels. The combined received signal can be written as [7]

\[ r = \sum_{q=0}^{U-1} h_q \ast (a_{q,TX}x_q) + z \]  

where \( r \) is the received OFDMA symbol, \( h_q \) is the \( D \times 1 \) channel impulse response vector, \( x_q \) is the \( N \times 1 \) transmitted OFDMA symbol, \( z \) denotes the AWGN effect and \( \ast \) denotes the convolution operation. \( a_{q,TX} \) denotes an \( N \times N \) matrix that contains \( \exp(j\phi_{q,TX}) \) in the diagonal, where \( \phi_{q,TX} \) is the \( N \times 1 \) PN realization of the \( q \)th user. The symbol \( m \) that denotes the indexing of an OFDMA symbol is omitted for clarity. In this scenario, the receiver PN is omitted for simplicity since the oscillators of the transmitters are anyway of lower quality than the ones at the base-station receivers. Finally, the inter-OFDM-symbol interference in (5) has been omitted since we assumed the use of a cyclic prefix with a length that exceeds that of the maximum channel delay spread.

In this paper, a contiguous subcarrier assignment is applied to the OFDMA uplink scenario performed in the following way: three users share the OFDMA symbol, and since both users at the edges will contribute in the PN induced spread on the center user’s subcarriers, the study focuses on the center user. The adjacent users also suffer from the PN spread, but the study is done from the point-of-view of the center user for the simplicity of the presentation. In the actual link, everything carried out for the center user is naturally carried out for all the users. The pilots for each user span the frequency range allocated for each user and do not crossover to the bands of the other users in an OFDMA symbol. A schematic diagram showing a simplified example of the subcarrier arrangement of the applied OFDMA uplink scenario is depicted in Fig. 1. A block diagram showing the OFDMA transmission scheme and the way the different OFDMA users add up at the receiver is depicted in Fig. 2.

III. IUI ESTIMATION ALGORITHM

The algorithm of estimating the PN induced spectral spread from users’ signals on top of the other users’ signals deploys the use of OFDM PN estimation techniques. The used PN estimation techniques are the CPE and ICI estimation techniques studied in [1] and [2] respectively. In this paper, the estimation of the PN induced spectral spread is performed in the following sequence. First, the CPE of an OFDMA symbol is estimated along with the channel effect. Next, the CPE and the channel effect are mitigated and the data symbols of OFDMA users are detected. The detected symbols are then used in the estimation of the ICI’s spectral components that can be used in the estimation of the spectral spread that eventually causes IUI. By applying the PN estimation algorithms in the above mentioned manner, we can obtain the frequency domain PN complex exponential \( J_{m,q} \) for each user. These are then used in the estimation of the IUI as follows. Considering the transmission scenario depicted in Fig. 1, three OFDMA users share the symbol. For simplicity, it is assumed that each user has an equal number of subcarriers denoted by \( M \). To estimate the PN induced spread caused by the edge users on the center one, the following equations are applied

\[ \hat{S}_{l,k}(m) = \bar{R}_{l,M+k}(m) \sum_{k=0}^{M} \hat{S}_{l,k}(m)\delta_{l,k-M}(m), \]  

\[ \hat{S}_{r,k}(m) = \bar{R}_{r,2M-k-1}(m) \sum_{k=2M+1}^{M} \hat{S}_{r,k}(m)\delta_{r,k-2M}(m), \]

where \( \hat{S}_{l,k}(m) \) are the estimated spread at the edges of the center user on the right side (from the user on the right side), \( \hat{S}_{l,k}(m) \) is the estimated spread at the edges of the center user on the left side (from the user on the left side). \( \bar{R}_{l}(m) \) are the detected symbols after ICI compensation of the left user, \( \bar{R}_{r}(m) \) are the detected symbols after ICI compensation of the right user, \( \bar{R}_{c}(m) \) is the estimated center-most frequency components of the right user and \( \bar{R}_{l}(m) \) is the estimated center-most frequency components of the left user. Finally, \( \bar{R}_{c}(m) \) are the channel estimates of the right and left users respectively where their indices overlap with the center user indices. The number of the estimated PN induced spread components is limited to \( u \) samples which is the number of side bins estimated in the ICI estimation.

After the IUI induced spectral spread is estimated, its removal from the center user is performed by applying the following equations

\[ R_{c,k} = \sum_{k=0}^{u} \bar{R}_{c,k}(m), \]  

\[ R_{c,M-k} = \sum_{k=0}^{u} \bar{R}_{c,M-k}(m). \]

where \( \bar{R}_{c} \) is an \( M \times 1 \) vector of the IUI mitigated received OFDMA subcarriers and \( \bar{R}_{c} \) is an \( M \times 1 \) vector of the received OFDMA subcarriers of the center user. Practically, \( \bar{S}_{l}(m) \) and \( \bar{S}_{r}(m) \) have \( u \) elements so only \( u \) components at the band edges of the center user are mitigated from the spread caused by the edge users. In this paper, we focus on mitigating the spread caused by the edge users on the center one for simplicity, while this method can be applied generally on any user regardless of his position in an OFDMA symbol.

IV. SIMULATIONS PARAMETERS AND RESULTS

This section provides the parameters used in the simulations. Additionally, it depicts the results of the simulations and highlights the
advantages of the proposed IUI mitigation algorithm when implemented.

A. Simulation Parameters
The simulations in this paper follow a contiguous subcarrier assignment for each user. Three users share an OFDMA symbol as depicted in Fig. 1. The user whose performance is studied in various cases is the center one since it suffers from the spread caused by both edge users and is therefore the most interesting from the IUI point-of-view. 36 of the total 1200 active subcarriers are assigned to the center user and the rest of the subcarriers are equally assigned to the edge users. This was done to give the studied center user a relatively narrow bandwidth, in which case the relative effect of IUI is significant. In this case we see how well the IUI mitigation method really performs. In practice, users can have even narrower bands, where the effect is clearer. The used FFT size is 2048, so the rest of the available subcarriers are null and act as a guard band at the edges of the OFDMA symbol. The sampling time and frequency are 0.0651 μs and 15.36 MHz respectively which were taken from the LTE specifications [9]. The used channel model is the extended ITU-R Vehicular A multipath channel [10], and the cyclic prefix length is 2.6042 μs (40 samples) that is larger than the used channel model delay spread. Each of the OFDMA users experiences the channel effect independently and it varies for each OFDMA symbol. The pilot subcarriers assignment was done for every fourth subcarrier. The transmitter generates 16-QAM symbols with Gray coding and then modulates them using the IFFT algorithm. Additionally, the specifications regarding the transmission of the OFDMA waveforms were checked to insure that the transmission scenario abides by the specifications of the LTE system published in [9]. The two requirements to be met are the Error Vector Magnitude (EVM) and the side lobe powers of the transmitted channel. Following these requirements, it was deduced that the FRO 3-dB bandwidth β should not exceed roughly 50 Hz when transmitting an OFDMA symbol in the uplink scenario implemented in this paper.

B. Simulation Results and Analysis
In the simulations, the FRO 3-dB bandwidth (β) is 50 Hz, the power difference between the edge users and the center one is 10 dB and the SNR is given relative to the center user, so each user has a different SNR since the additive noise is white over the whole OFDMA symbol band. The legends in the figures mean the following: CPE means that the CPE is estimated and mitigated for the center user, while CPE-IUI denotes the same case but IUI is also mitigated for w=3. The same explanation applies for ICI and ICI-IUI but with the application of the ICI algorithm studied in [2] instead of the plain CPE algorithm. The ICI estimation technique is iterated three times for all users. Finally, two reference curves are added. “Full PN Knowledge” means that the PN of the edge and center users are known at the receiver, so the in-band PN and the out-of-band PN induced spread, or IUI, are fully known and compensated, while “Partial PN Knowledge” means that only the 2w+1=7 centermost ICI frequency components are known which includes the ICI DC component along with 3 spectral components at each side.

At this stage, the channel effect is considered to be known at the receiver to focus on the PN effect. Since the PN induced spread stretches outside the bands allocated to each of the users, the channel at these locations must be estimated. In this paper, no out-of-band pilot data is inserted to estimate the channel beyond a certain user’s band, but a simple duplication of the last known channel frequency response samples at the edges of the user causing the spread is done. From the results is it clear that this simple method proved to work very well.

Fig. 3. BER vs. SNR analysis for an AWGN channel. The power difference between the edge users and the center one is 10 dB and the PN 3-dB bandwidth (β) is 50 Hz and equal for all users.

Fig. 4. BER vs. SNR analysis for the extended ITU-R Vehicular A multipath channel. The power difference between the users and the center one is 10 dB and the PN 3-dB bandwidth (β) is 50 Hz and equal for all users.

Fig. 5. BER vs. PN 3-dB bandwidth (β) for an AWGN channel. β is equally varied for the edge users while fixed for the center user at 50 Hz. The center user SNR is fixed to 25 dB and the power difference between the edge users and the center one is 10 dB.
characterization and compensation of an OFDMA system. It creates IUI and its estimation and mitigation are considered in this paper. Estimating the IUI and mitigating its effect showed a significant gain over the conventional CPE and ICI estimation algorithms. For a contiguous OFDMA subcarrier assignment, there was no need for inserting out-of-band pilots to estimate the channel effect beyond a user’s bandwidth, and a simple interpolation was enough to estimate it to obtain a good performance. The BER performance deteriorates when the power difference between the users and the PN 3-dB bandwidth are increased for all of the applied algorithms.

**References**


