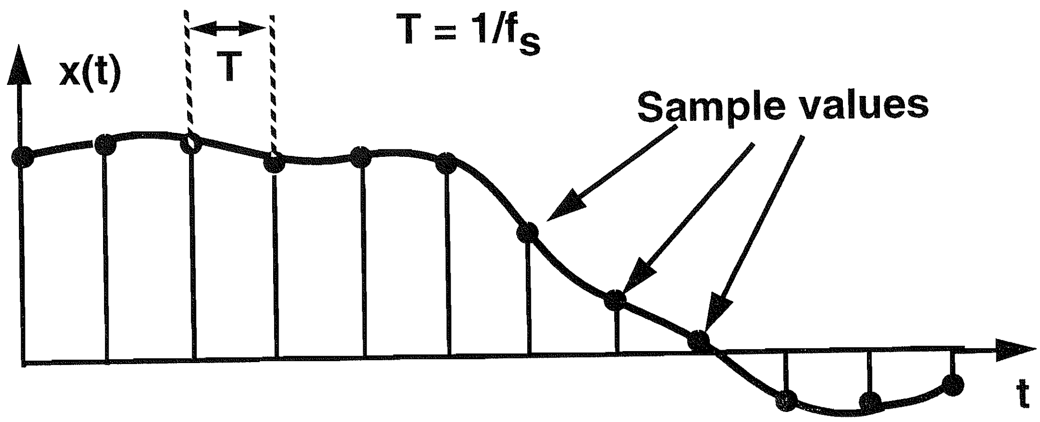
80558 MULTIRATE SIGNAL PROCESSING

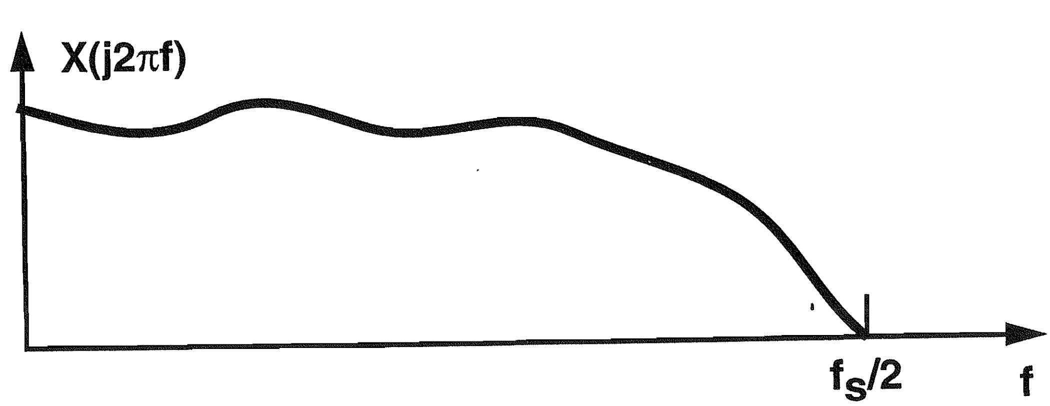
Part I: Basics and Motivation

- The purpose of this part is to give some motivation for multirate digital signal processing.
- First, the starting point for processing continuous-time signals with the aid of digital signal processing is considered.
- Second, the need for sampling rate alteration is discussed.
- Third, the two types of sampling rate alteration are considered:
    - Sampling rate reduction, called *decimation*.
    - Sampling rate increase, called *interpolation*.
- Fourth, some applications are considered.

Starting Point for Digital Signal Processing

Sampling theorem: A continuous-time signal can be reconstructed from its **sample values** if **the sampling frequency** $f_s = 1/T$ (T is the sampling period) is at least two times the highest frequency component of the signal, that is, $X(j2\pi f) = 0,\ f > f_s/2$. See the figures shown below.





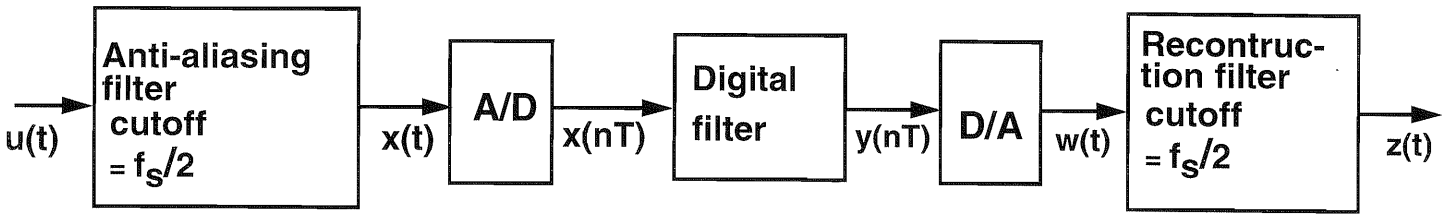
Processing of a Continuous-Time Signal with the Aid of a Digital Filter

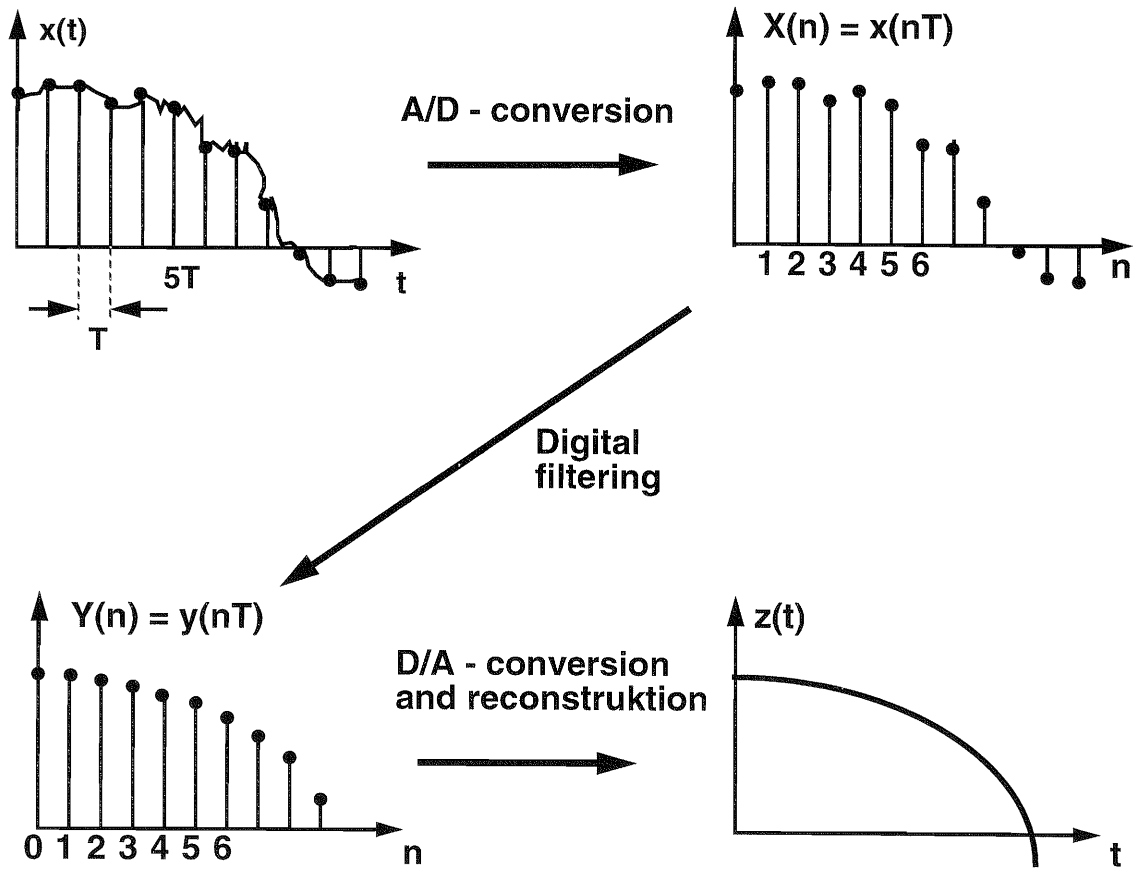
- Based on the sampling theorem, the processing of a continuous-time signal can be performed with the aid of the discrete-time system as shown on the next page.
- In order to satisfy the conditions of the sampling theorem, the continuous-time signal has to band-limited to the frequency range $-f_s/2 \leq f \leq f_s/2$ using an anti-aliasing filter.
- Ideally, the output signal can be generated from the output samples $y(nT) \equiv Y(n)$ with the aid of the following sinc-interpolation:

$$z(t) = \sum_{k=-\infty}^{\infty} y(nT) \frac{\sin[\pi(t - nT)/T]}{\pi(t - nT)/T}. \tag{1}$$

- In practice, an analog reconstruction filter is used for approximating this interpolation.
- In the frequency domain, this means that from the periodic response of the discrete-time output signal only the baseband frequencies $-f_s/2 \leq f \leq f_s/2$ are preserved.

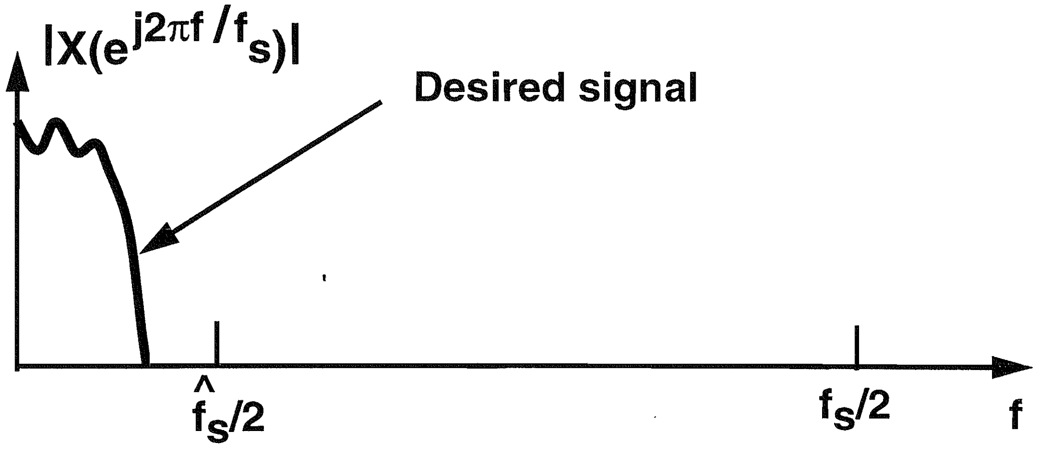
Processing of a Continuous-Time Signal with the Aid of a Digital Filter





Needs for Alterating the Sampling Rate f_s

• There exist several situations where the signal of interest is staying in a frequency range that is very small compared to half the sampling rate $f_s/2$ as shown in the following figure.



• In the case of this figure, the sampling rate $\widehat{f_s} = f_s/3$ could be used.

• There are several advantages of using $\widehat{f_s}$ as a sampling rate, instead of f_s, and to make it as small as possible.

Advantage 1 of Using a Lower Sampling Rate: The number of samples is reduced by a factor of $f_s/\widehat{f_s}$ so that the processing workload is significantly reduced.

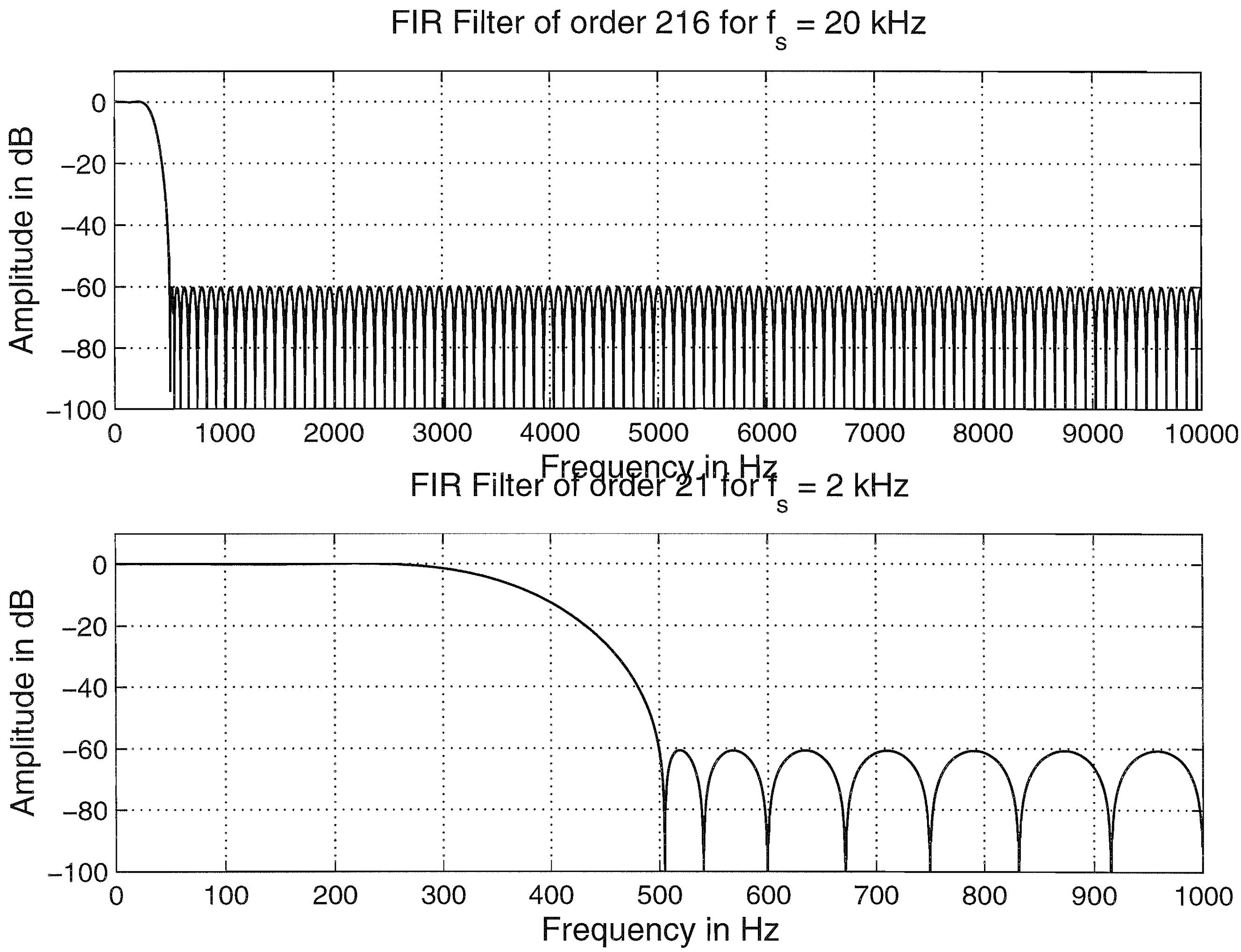
- For instance, if in the case of the figures on page 2, $X(j2\pi f) = 0$, $f > (f_s/2)/3$, then sampling rate $\widehat{f_s} = f_s/3$ can be used.

- This means that the corresponding sampling period $\widehat{T} = 1/\widehat{f}_s = 3T$. In this case, only every third sample of the first figure are needed for carrying the information of the continuous-time signal!

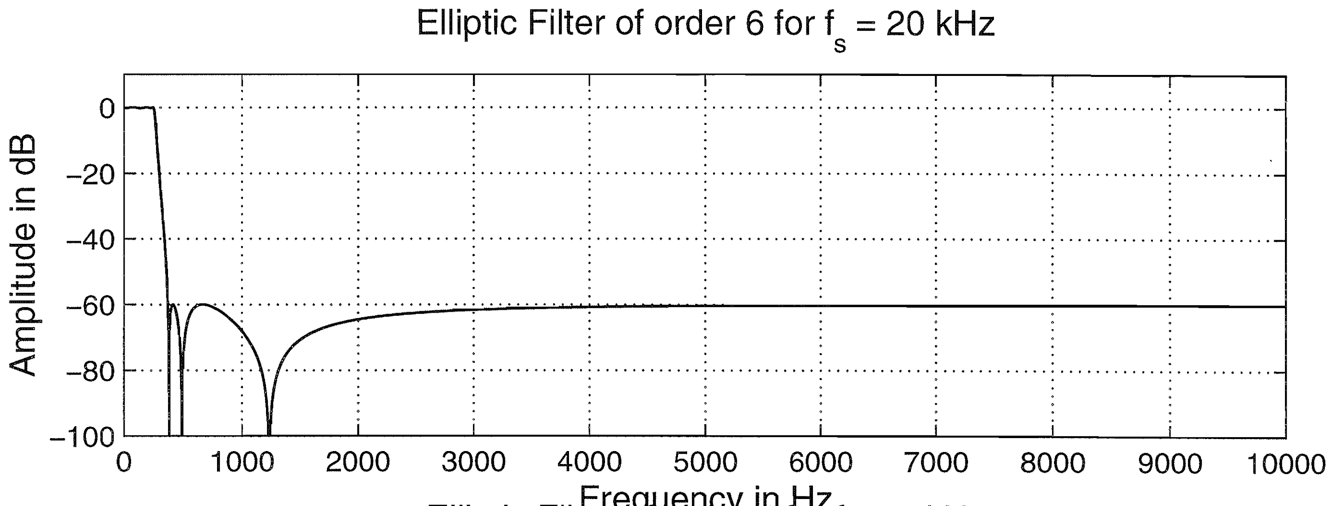
Advantage 2 of Using a Lower Sampling Rate: The discrete-time systems become easier to implement.

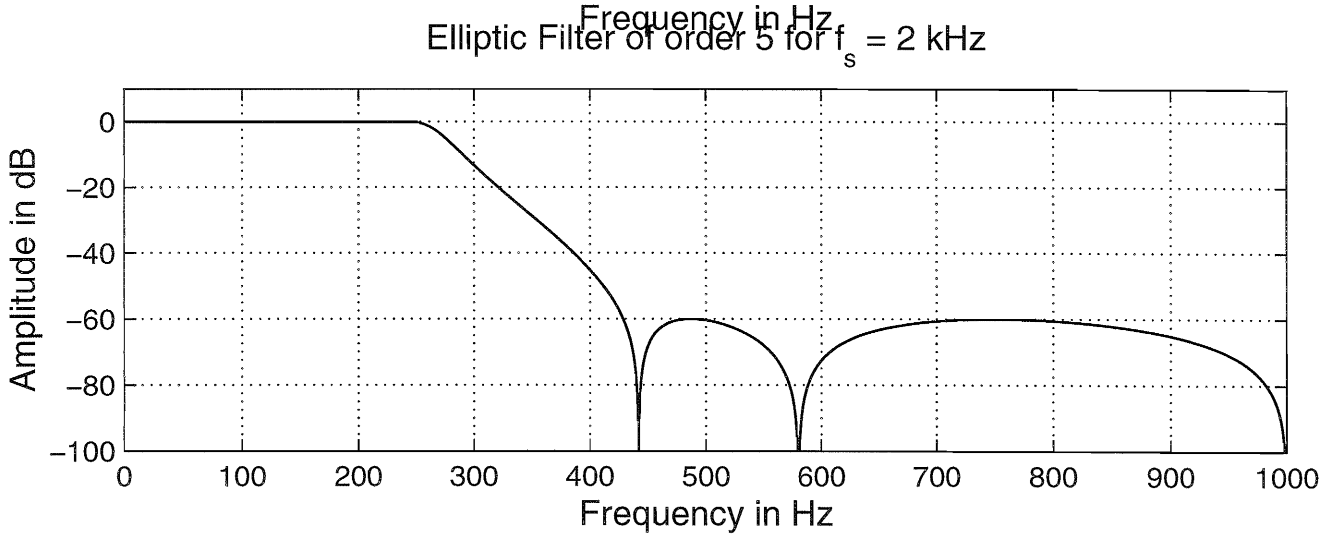
- As shown on the next page, the order of an FIR filter reduces approximately by 10 if the sampling rate is reduced by this factor.
- Taking into account the fact that also the number of data samples is only one tenth, the overall saving is approximately 10·10=100!
- Pages 7 and 8 illustrate what happens in the case of elliptic IIR filter.
- For these filters, the order either remains the same or decreases by one when using a lower sampling rate.
- The main advantage lies in the the fact that the poles of the filter implemented using a lower sampling rate are further away from the unit circle.
- This means that the finite wordlength effects are significantly milder: significantly fewer bits are required for both the data and coefficient respresentations.

Linear-Phase FIR Filter with Passband and Stopband Edges at 250 Hz and 500 Hz and Passband and Stopband Ripples of 0.01 and 0.001 for the Amplitude Response



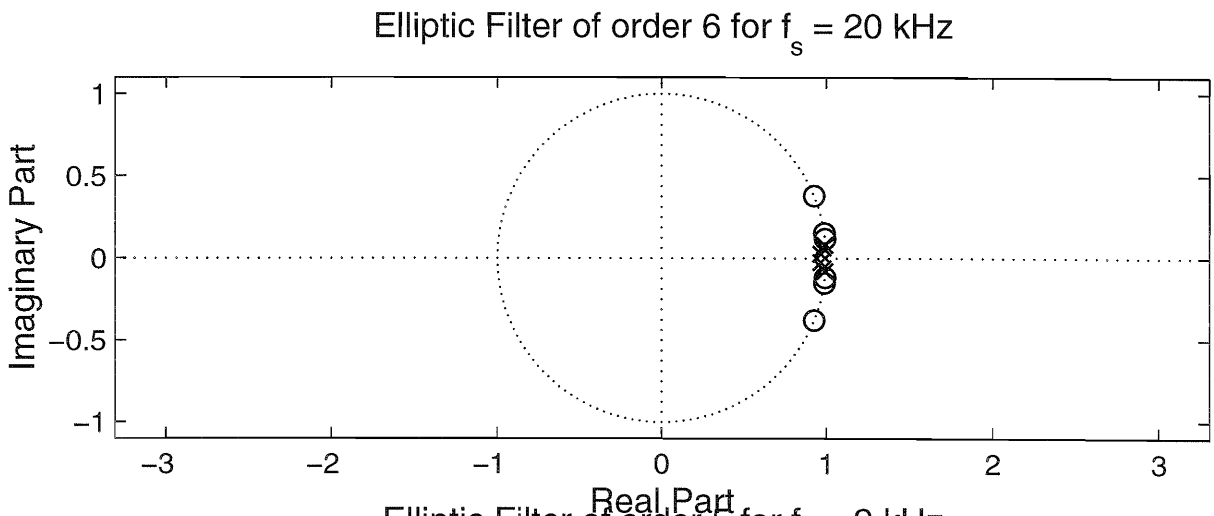
Elliptic Filter with Passband and Stopband Edges at 250 Hz and 500 Hz and Passband and Stopband Ripples of 0.2 dB and 60 dB

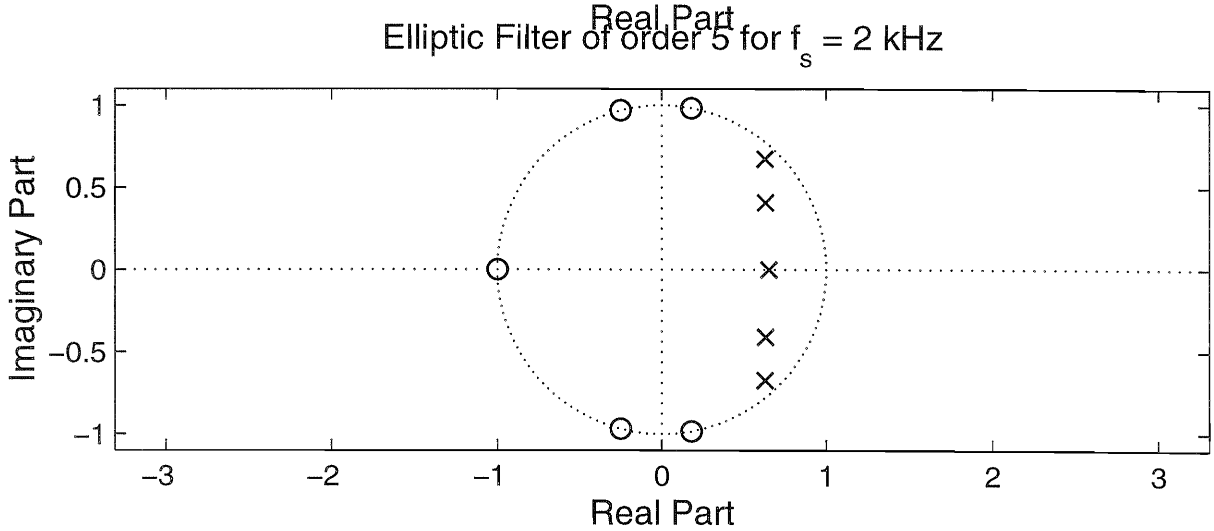




Elliptic Filter with Passband and Stopband Edges at 250 Hz and 500 Hz and Passband and Stopband Ripples of 0.2 dB and 60 dB

- Since the filter poles for $f_s = 2$ kHz are not so close to the unit circle, the coefficient sensitivity as well as the output noise are much lower than for $f_s = 20$ kHz.



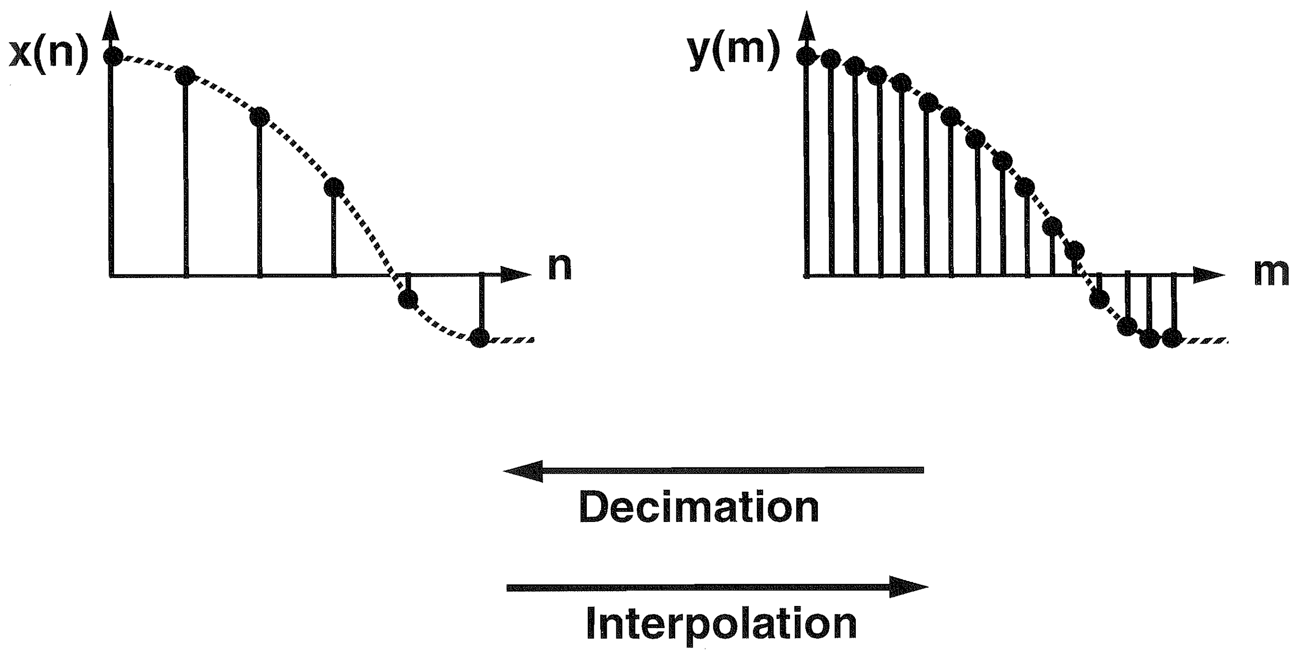


Needs for Alterating the Sampling Rate f_s

- In addition to the above example, there are numerous other applications where it is advantageous or even necessary to change (reduce or increase) the sampling rate, as will be seen later on in this course.
- In our example case, the sampling rate of the signal of the system of Page 4 could be reduced according to the characteristics of our input continuous-time signal.
- However, in most applications, there exist signals having different bandwidths.
- Therefore, it is preferred to study how to change the sampling rate directly in the digital domain.

The Two Basic Types for Sampling Rate Alteration

- There exist two types of sampling rate alteration, namely decimation and interpolation.
- As shown below, in the case of decimation, the number of samples is reduced.
- This means that the sampling period is increased and the sampling rate is decreased.
- In the case of interpolation, the number of samples is increased.
- This means that the sampling period is reduced and the sampling rate is increased.



Decimation by an Integer Factor M

• When reducing the sampling rate by an integer factor of M, the overall system is constructed as shown on the next page.

• The first step is to filter the input signal $x(n)$ with a transfer function $H(z)$ so that the z- and Fourier transforms of the filtered signal $w(n)$ are given by

$$W(z) = H(z)X(z) \tag{2a}$$

and

$$W(e^{j2\pi f/f_s}) = H(e^{j2\pi f/f_s})X(e^{j2\pi f/f_s}), \tag{2b}$$

where f_s is the input sampling rate.

• The second step is to pick up every Mth sample of $w(n)$ to form the output signal $y(m)$ (the arrow downwards followed by M means this operation).

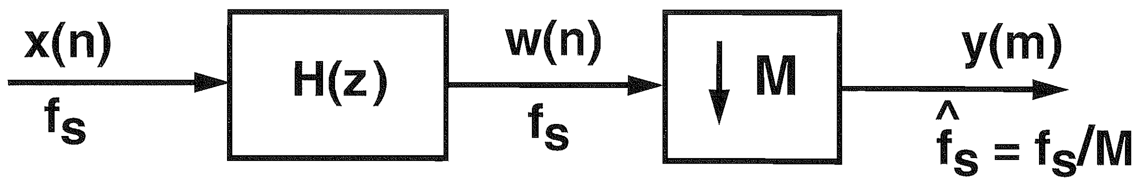
• $y(m)$ is thus related to $w(n)$ via

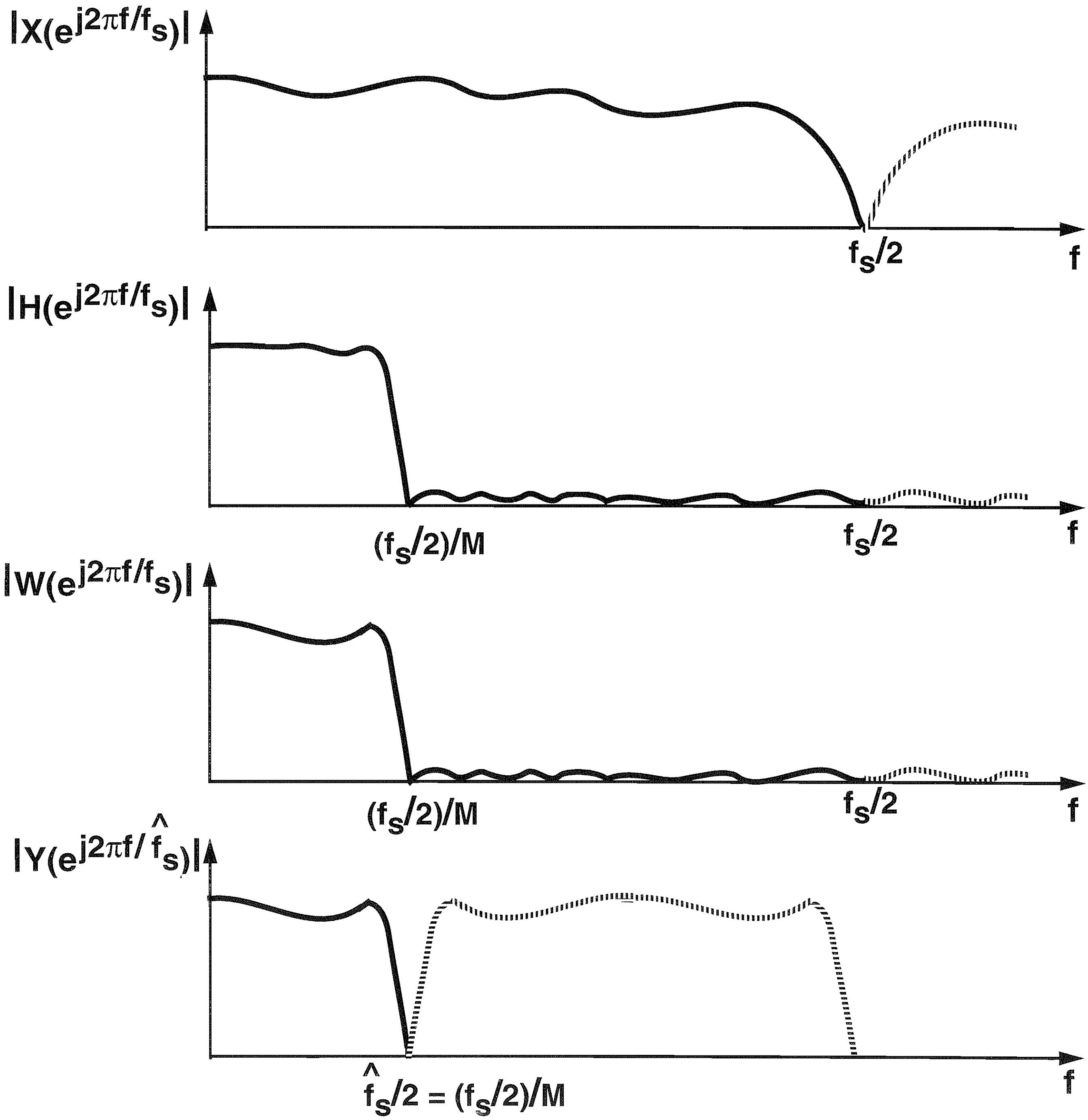
$$y(m) = w(mM) \tag{3}$$

so that $y(0) = w(0)$, $y(1) = w(M)$, $y(2) = w(2M)$ and so on.

• The sampling rate of $y(m)$ is thus $\widehat{f_s} = f_s/M$ and the new baseband is $[0,\ \widehat{f_s}/2] = [0,\ (f_s/M)/2]$.

Block Diagram for Decimation by an Integer Factor M





• It can be shown that the z- and Fourier transforms of the decimated signal $y(m)$ are given by

$$Y(z) = \frac{1}{M} \sum_{k=0}^{M-1} W(z^{1/M} e^{j2\pi k/M}) \qquad (4a)$$

and $(z = e^{j2\pi f/(f_s/M)})$

$$Y(e^{j2\pi f/(f_s/M)}) = \frac{1}{M} \sum_{k=0}^{M-1} W(e^{j(2\pi/f_s)(f+kf_s/M)}). \qquad (4b)$$

• In the above equation, the term for $k = 0$, that is, $W(e^{j2\pi f/f_s})$ is the frequency component staying in the new baseband region $|f| \leq \widehat{f_s}/2 = (f_s/M)/2$ before the sampling rate reduction.

• The other terms $W(e^{j(2\pi/f_s)(f+kf_s/M)})$ for $k = 1, 2, \cdots,$ $M-1$ are the undesired components **aliasing** from $(k-1/2)f_s/M \leq f \leq (k+1/2)f_s/M$ into the new baseband $|f| \leq \widehat{f_s}/2 = (f_s/M)/2$.

• The aliasing or overlapping of several terms can be avoided by requiring that before decimation $W(e^{j2\pi f/f_s})$ is practically nonzero only in the new baseband $|f| \leq \widehat{f_s}/2 = (f_s/M)/2$. In this band, it is then true that $Y(e^{j2\pi f/(f_s/M)}) \approx W(e^{j2\pi f/f_s})$.

• In the ideal case, it is desired that $Y(e^{j2\pi f/(f_s/M)}) \approx e^{-j2\pi\alpha/f_s} X(e^{j2\pi f/f_s})$, that is, $y(m)$ is a delayed and decimated version of $x(n)$ in the frequency band $|f| \leq \widehat{f_s}/2$ $= (f_s/M)/2$. (Note that some delay is always needed when filtering a signal.)

• This is achieved by designing $H(z)$ to satisfy

$$H(e^{j2\pi f/f_s}) \approx \begin{cases} e^{-j2\pi\alpha f/f_s} & \text{for } f \leq (f_s/2)/M \\ 0 & \text{for } (f_s/2)/M \leq f \leq (f_s/2). \end{cases} \tag{5}$$

• If the phase characteristics is not of importance, then it is desired that $|Y(e^{j2\pi f/(f_s/M)})| \approx |X(e^{j2\pi f/f_s})|$ for $|f| \leq \widehat{f}_s/2 = (f_s/M)/2$.

• In this case, it is required that

$$|H(e^{j2\pi f/f_s})| \approx \begin{cases} 1 & \text{for } f \leq (f_s/2)/M \\ 0 & \text{for } (f_s/2)/M \leq f \leq (f_s/2). \end{cases} \tag{6}$$

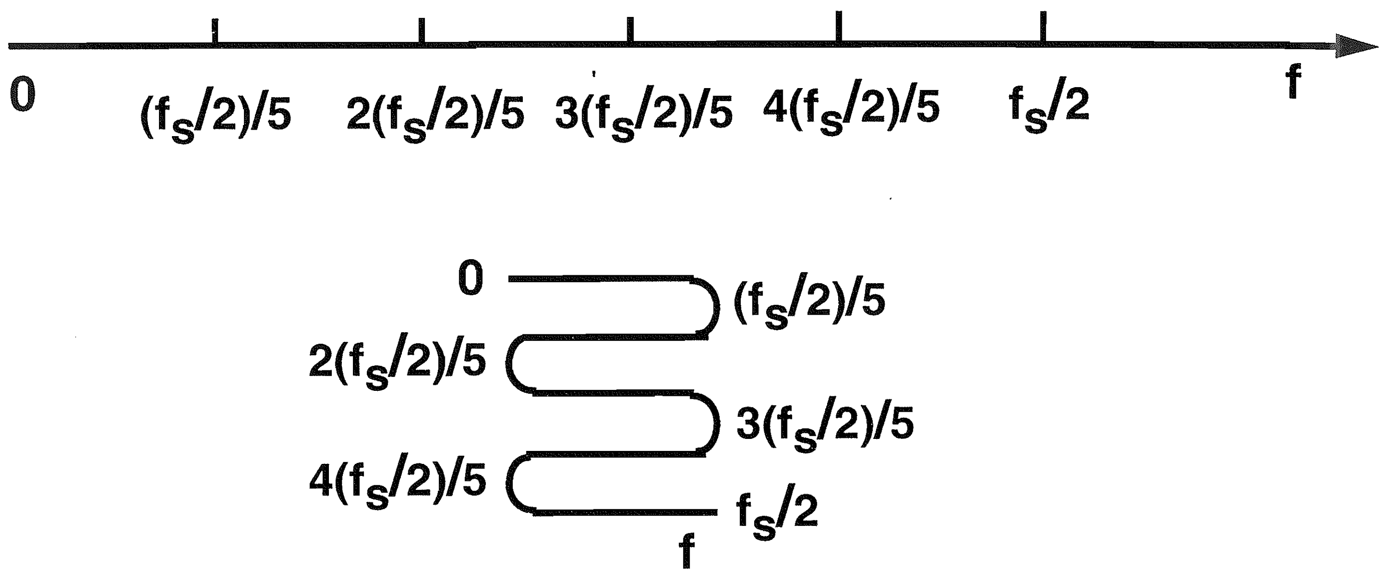
Comment 1: In Eqs. (4) the multiplier $1/M$ comes just from the use of the mathematics. The signal levels in the time domain remain the same.

Comment 2: In Eq. (4a), instead of $e^{j2\pi k/M}$, $e^{-j2\pi k/M}$ is usually used. In this case, $(f + kf_s/M)$ becomes $(f - kf_s/M)$ in Eq. (4b). The basic reason for our selection is the fact that the explanation of aliasing becomes more straightforward.

• The role of the filter with transfer function $H(z)$ is thus similar to the analog anti-aliasing filter: the signal components of the input sequence $x(n)$ outside the range $-\widehat{f}_s/2 \leq f \leq \widehat{f}_s/2$ should be attenuated in order to avoid aliasing.

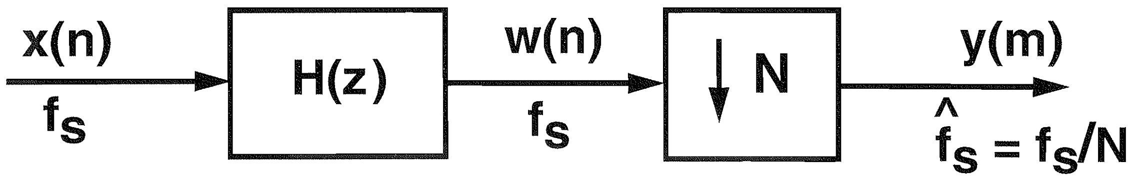
A Simple Way of Checking How the Frequency Components Are Aliasing

The following figure illustrates in the $M = 5$ case how the components from the range $(f_s/2)/M < f \leq f_s/2$ are aliasing into the new baseband $0 \leq f \leq (f_s/2)/M$.



Requirements for $H(z)$ in the Decimation Case

- There exist three ways of stating the criteria for $H(z)$.
- In order to make the consideration of interpolators the same, $N \equiv M$ and the following structure are used:



- In terms of the angular frequency $\omega = 2\pi f/f_s$, they can be stated as

$$1 - \delta_p \leq |H(e^{j\omega})| \leq 1 + \delta_p \quad \text{for} \quad \omega \in [0, \alpha\pi/N], \qquad (7a)$$

where $\alpha < 1$, and

$$|H(e^{j\omega})| \leq \delta_s \quad \text{for} \quad \omega \in \Omega_s, \qquad (7b)$$

where

$$\Omega_s = \begin{cases} [\pi/N, \ \pi] & \text{for Case A} \\ \displaystyle\bigcup_{k=1}^{\lfloor N/2 \rfloor} [\frac{(2k-\alpha)\pi}{N}, \ \min(\frac{(2k+\alpha)\pi}{N}, \ \pi)] & \\ & \text{for Case B} \\ [(2-\alpha)\pi/N, \ \pi] & \text{for Case C.} \end{cases} \qquad (7c)$$

- Similarly, in terms of the 'real' frequency, these criteria can be stated as

$$1-\delta_p \leq |H(e^{j2\pi f/f_s})| \leq 1+\delta_p \quad \text{for} \quad \omega \in [0, \alpha(f_s/2)/N], \tag{8a}$$

where $\alpha < 1$, and

$$|H(e^{j2\pi f/f_s})| \leq \delta_s \quad \text{for} \quad \omega \in X_s, \tag{8b}$$

where

$$X_s = \begin{cases} [(f_s/2)/N,\ \ f_s/2] & \text{for Case A} \\ \bigcup_{k=1}^{\lfloor N/2 \rfloor} [\frac{(2k-\alpha)f_s/2}{N},\ \min(\frac{(2k+\alpha)f_s/2}{N},\ \ f_s/2)] & \\ & \text{for Case B} \\ [(2-\alpha)(f_s/2)/N,\ \ f_s/2] & \text{for Case C.} \end{cases} \tag{8c}$$

• In all the cases, the signal is preserved in the passband region $[0, \alpha(f_s/2)/N]$ with $\alpha < 1$.

• In Case A, all the components aliasing into the new baseband $[0, (f_s/2)/N]$ are attenuated.

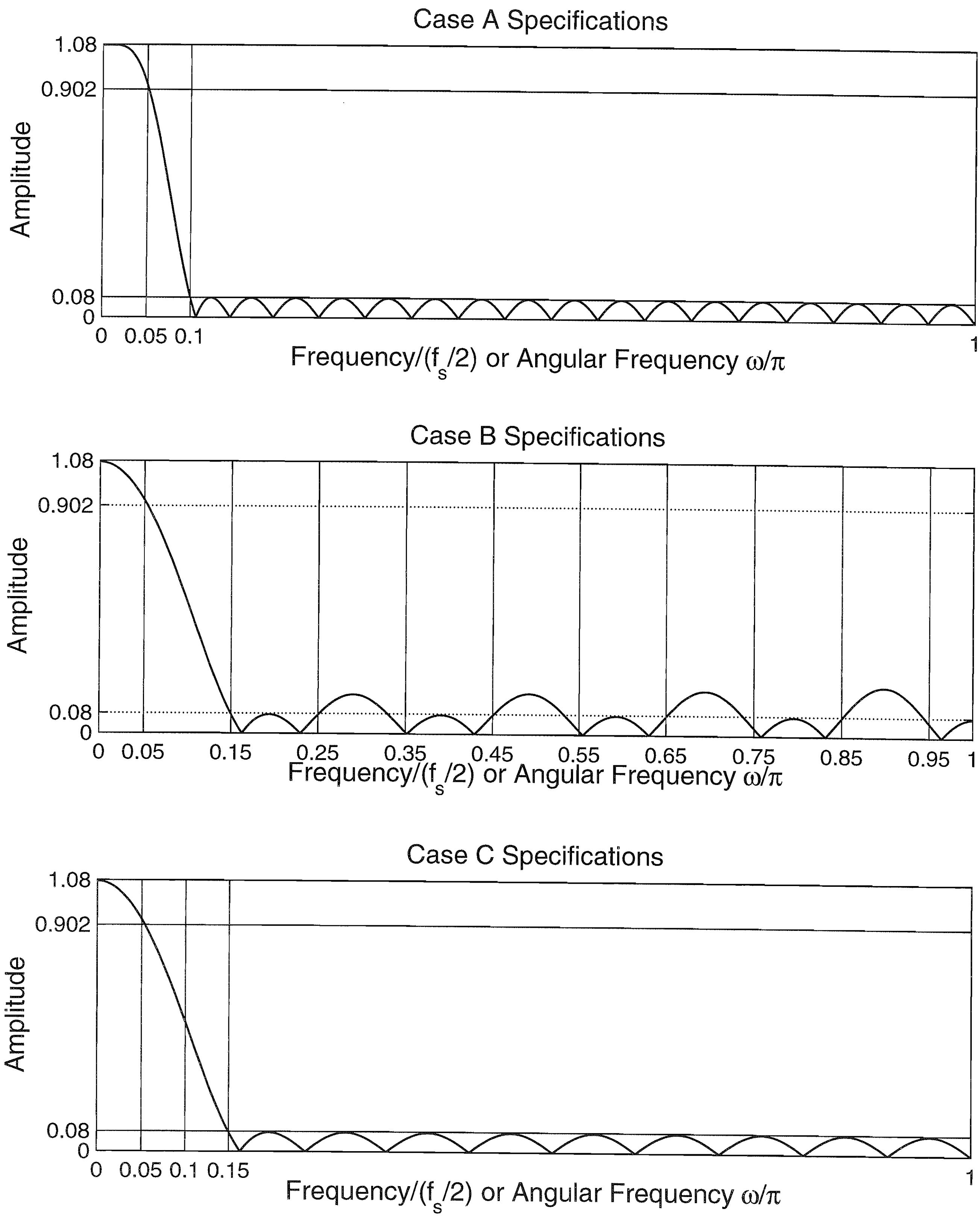
• In Case B, all the components aliasing into the passband $[0,\ \alpha(f_s/2)/N]$ are attenuated, but aliasing is allowed into the transition band $[\alpha(f_s/2)/N,\ (f_s/2)/N]$.

• In Case C, aliasing is allowed into the transtion band $[\alpha(f_s/2)/N,\quad (f_s/2)/N]$ only from the band $[(f_s/2)/N,\ (2-\alpha)(f_s/2)/N]$.

• Case B and C specifications can be used, for instance, in audio applications when f_s/N=44.1 kHz and $\alpha = 0.907$. In this case the transition band is between 20 kHz and 22.05 kHz, that is, outside the frequency range a human ear is able to hear.

- Page 20 shows the responses for $N = 10$, $\alpha = 0.5$, and $\delta_p = \delta_s = 0.08$ in these three cases. The passband region is thus in terms of the 'real' frequency $[0,\ 0.05(f_s/2)]$ and in terms of the angular frequencies $[0,\ 0.05\pi]$.
- For Case A, the stopband region is in terms of the angular frequencies $[\pi/10,\ \pi]$ and in terms of the 'real' frequency $[(f_s/2)/10,\ f_s/2]$.
- For Case B, the stopband region is in terms of the angular frequency the union of the bands $[1.5\pi/10,\ 2.5\pi]$, $[3.5\pi/10,\ 4.5\pi]$, $[5.5\pi/10,\ 6.5\pi]$, $[7.5\pi/10,\ 8.5\pi]$, and $[9.5\pi/10,\ \pi]$.
- In terms of the 'real' frequency, the stopband region is the union of the bands $[1.5\pi/10,\ 2.5\pi]$, $[3.5(f_s/2)/10,\ 4.5(f_s/2)]$, $[5.5(f_s/2)/10,\ 6.5(f_s/2)]$, $[7.5(f_s/2)/10,\ 8.5(f_s/2)]$, and $[9.5(f_s/2)/10,\ f_s/2]$.
- For Case C, the stopband region is in terms of the angular frequency $[1.5\pi/10,\ \pi]$ and in terms of the real frequencies $[1.5(f_s/2)/10,\ f_s/2]$.

Example Case A, Case B, and Case C Specifications: $N = 10$, $\alpha = 0.5$, and $\delta_p = \delta_s = 0.08$



Interpolation by an Integer Factor L

- When increasing the sampling rate by an integer factor L, the overall system is constructed as shown on the next page.
- In the first step, $L-1$ zero-valued samples are inserted between the existing input samples (the arrow upwards followed by L means this operation).
- Hence,

$$w(m) = \begin{cases} x(m/L), & m = 0, \pm L, \pm 2L, ... \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

- The sampling rate of $w(m)$ is thus $\widehat{f_s} = Lf_s$ and the new baseband is $|f| \leq \widehat{f_s}/2 = Lf_s/2$, that is, L times that of the input signal.
- The z- and Fourier transforms of the interpolated signal $w(m)$ are given by

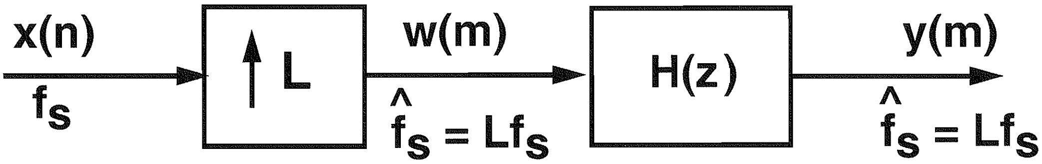
$$W(z) = X(z^L) \tag{10a}$$

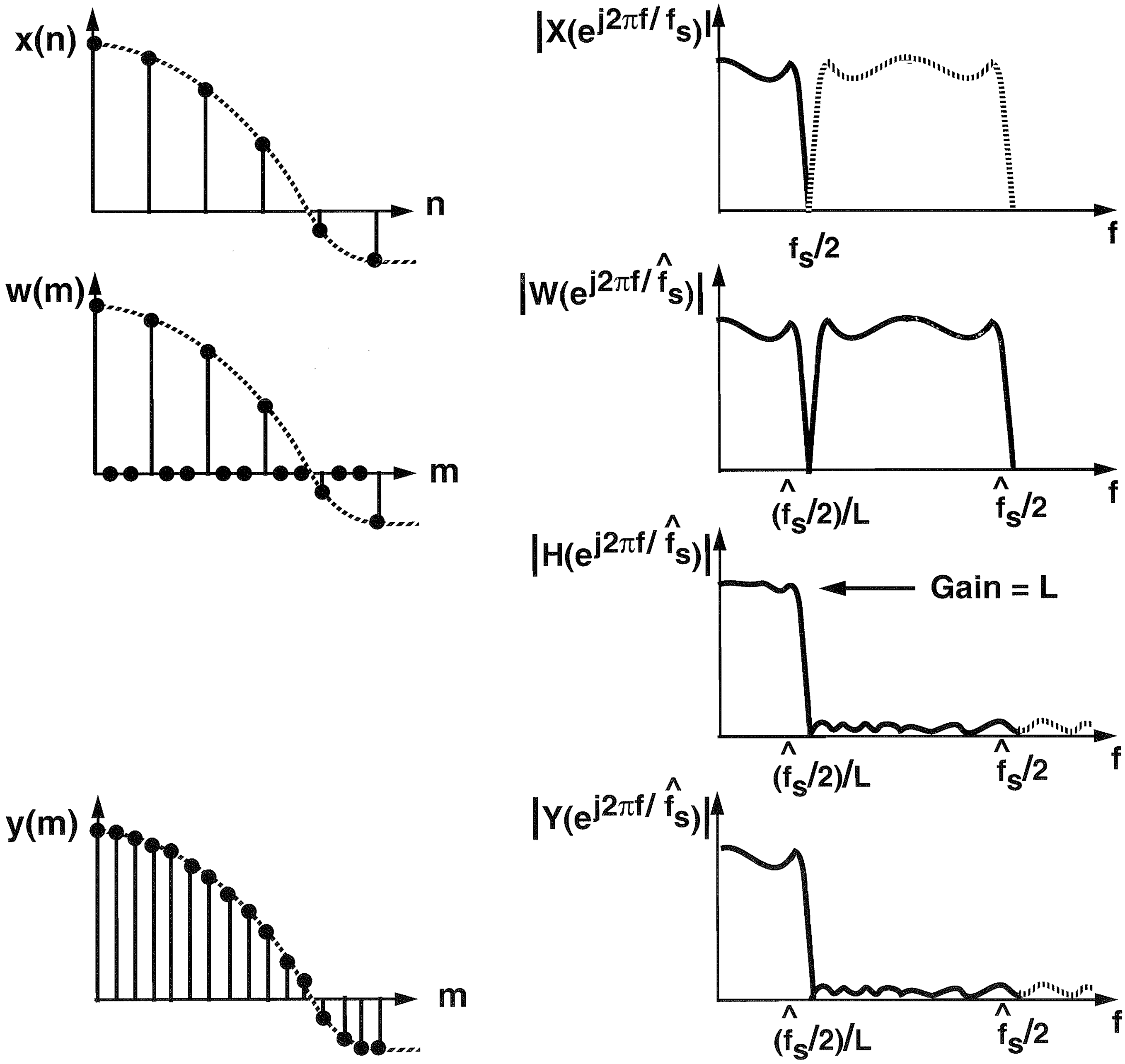
and

$$W(e^{j2\pi f/\widehat{f_s}}) = X(e^{j2\pi f/(\widehat{f_s}/L)}) = X(e^{j2\pi f/f_s}). \tag{10b}$$

- As illustrated on the next page, the Fourier transform of $w(m)$ contains in the increased basedband not only the baseband frequencies of $x(n)$ (i.e., $|f| \leq f_s$) but also images of the old baseband centered at $\pm f_s = \pm\widehat{f_s}/L$, $\pm 2f_s = \pm 2\widehat{f_s}/L$, $\cdots$.

Block Diagram for Interpolation by an Integer Factor L





- The role of the filter with transfer function $H(z)$ is thus to preserve the original baseband $|f| \leq f_s/2 = (\widehat{f}_s/2)/L$ and to attenuate the unwanted higher frequency components (images) in the new baseband $|f| \leq \widehat{f}_s/2 = Lf_s/2$, that is, the frequencies $(\widehat{f}_s/2)/L \leq |f| \leq \widehat{f}_s/2$.
- In the ideal case, it is desired that $Y(e^{j2\pi f/(f_s/M)})$ approximates closely $e^{-j2\pi\alpha/f_s}LX(e^{j2\pi f/f_s})$ for $|f| \leq (\widehat{f}_s/2)/L$ and zero elsewhere in the new increased baseband.
- In this case, $y(m)$ is a delayed and interpolated version of $x(n)$ in the frequency band $|f| \leq f_s/2 = (f_s/M)/2$.
- Note that the multiplier L is required to preserve the existing input samples in the time domain at the same level as well as raising the inserted zero-valued samples as 'interpolated' samples.
- This is achieved by designing $H(z)$ to satisfy

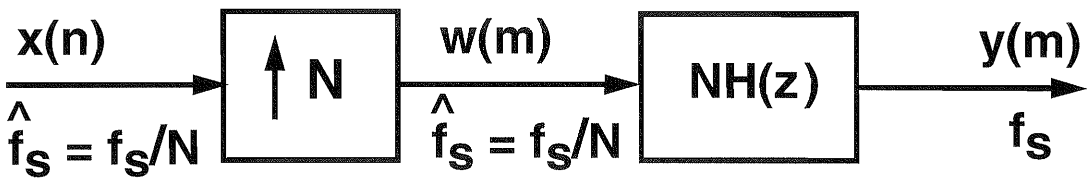
$$H(e^{j2\pi f/\widehat{f}_s}) \approx \begin{cases} Le^{-j2\pi\alpha/f_s} & \text{for } f \leq (\widehat{f}_s/2)/L \\ 0 & \text{for } (\widehat{f}_s/2)/L \leq f \leq \widehat{f}_s/2. \end{cases} \tag{11}$$

- If the phase characteristics is not of importance, then it is desired that

$$|H(e^{j2\pi f/\widehat{f}_s})| \approx \begin{cases} L & \text{for } |f| \leq (\widehat{f}_s/2)/L \\ 0 & \text{for } (\widehat{f}_s/2)/L \leq f \leq \widehat{f}_s/2. \end{cases} \tag{12}$$

Requirements for $H(z)$ in the Interpolation Case

• In order to state the criteria for $H(z)$ in a manner similar to the case of decimation we use the following structure:



• The main difference is that now the input and output sampling rates are f_s/N and f_s, instead of f_s and Lf_s. Furthermore, the interpolation ratio is N, instead of L.

• These modifications emphasize the **duality** between the decimator and interpolator designs.

• After designing the decimator for the integer decimation factor N, the corresponding interpolator can be generated as follows:

(a) Replace the decimation block after filtering by the transfer function $H(z)$ by the corresponding interpolation block before filtering by the transfer function $NH(z)$.

(b) Replace the input and output sampling rates f_s and f_s/N by f_s/N and f_s, respectively.

- Note that if the input signal satisfies for the input sampling rate of f_s/N $|X(e^{j2\pi f/(f_s/N)})| \approx 0$ for $\alpha(f_s/2)/N \leq |f| \leq (f_s/2)/N$, then the Case B or Case specifications, stated earlier in the decimation case, can be directly used for attenuating the extra images.

Sampling Rate Conversion by L/M with L and M Being Integers

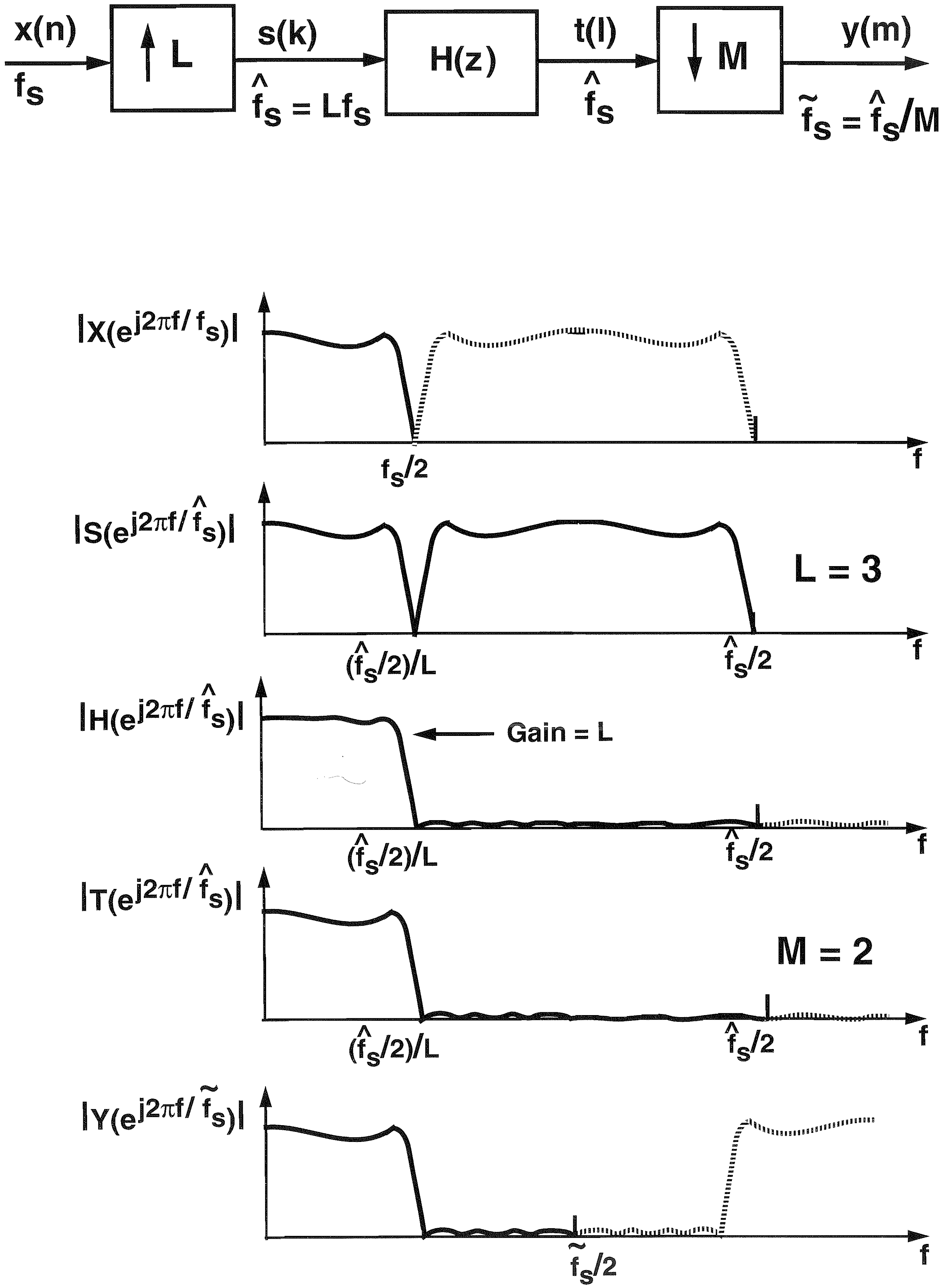
- Sampling rate conversion by L/M with both L and M being integers can be performed with the aid of a single filter.
- This is exemplified on Pages 28 and 29 in the $L = 3$ and $M = 2$ case and in the $L = 2$ and $M = 3$ case, respectively.
- Note that in the first case the sampling rate is increased by 3/2 and in the second case decreased by 3/2.
- In both cases, the sampling rate is first increased by L so that the resulting sampling rate is $\widehat{f_s} = Lf_s$.
- In the first case $L = 3 > M = 2$. Therefore, $H(z)$ acts like a filter for interpolation and satisfies

$$|H(e^{j2\pi f/\widehat{f_s}})| \approx \begin{cases} L & \text{for } |f| \leq (\widehat{f_s}/2)/L \\ 0 & \text{for } (\widehat{f_s}/2)/L \leq f \leq \widehat{f_s}/2. \end{cases} \quad (13)$$

- What is left is to reduce the sampling rate by $M = 2$, that is, to attenuate the signal components in the range $[(\widehat{f_s}/2)/2,\ \widehat{f_s}/2]$ in order to give the output sampling rate $\widetilde{f_s} = \widehat{f_s}/2 = (3/2)f_s$.
- As seen from Page 28, the filter with transfer function $H(z)$ has already attenuated these components.
- Therefore, every second sample at the output of this filter can be directly picked up without causing significant aliasing.

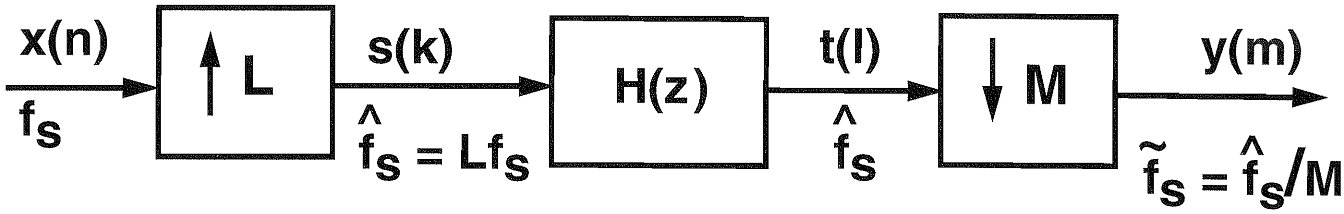
- In the $L = 2$ and $M = 3$ case, the role of $H(z)$ is different due to the fact that $M > L$.
- In this case, $H(z)$ has to attenuate the signal components in the range $[(\widehat{f}_s/2)/3,\ \widehat{f}_s/2]$ in order to give the output sampling rate $\widetilde{f}_s = \widehat{f}_s = (2/3)f_s$.
- Simultaneously, the images caused by interpolation are attenuated as well as a part of the original baseband, as illustrated on Page 29.
- Therefore, $H(z)$ acts like a filter for decimation with the execption that in the passband the desired value is L and satisfies

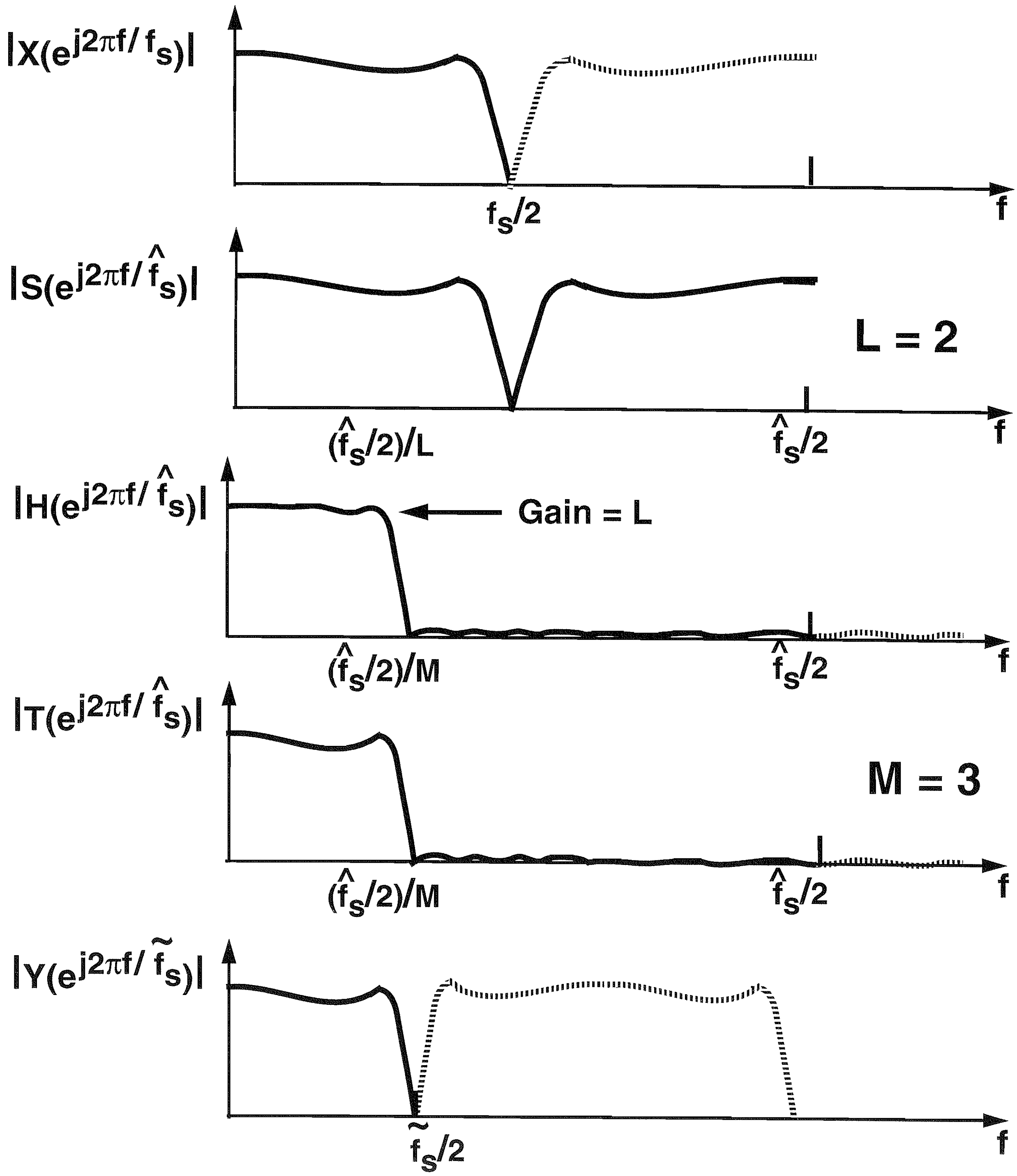
$$|H(e^{j2\pi f/\widehat{f}_s})| \approx \begin{cases} L & \text{for } |f| \leq (\widehat{f}_s/2)/M \\ 0 & \text{for } (\widehat{f}_s/2)/M \leq f \leq \widehat{f}_s/2. \end{cases} \tag{14}$$



x(n)
fs
L
s(k)
fs = Lfs
H(z)
t(l)
fs
M
y(m)
fs = fs/M
|X(ej2πf/fs)|
fs/2
f
|S(ej2πf/fs)|
L = 3
(fs/2)/L
fs/2
|H(ej2πf/fs)|
Gain = L
|T(ej2πf/fs)|
M = 2
|Y(ej2πf/fs)|
fs/2

Sampling Rate Conversion by L/M, $L = 2 < M = 3$





General Criteria for Sampling Rate Conversion by L/M with L and M being integers

- In the most general case, the sampling rate increase by a factor of L must be performed first resulting in the sampling rate $\widehat{f_s} = Lf_s$.
- The sampling rate $\widetilde{f_s} = \widehat{f_s}/M = (L/M)f_s$ is achieved by designing $H(z)$ to satisfy

$$|H(e^{j2\pi f/\widehat{f_s}})| \approx \begin{cases} L & \text{for } |f| \leq (\widehat{f_s}/2)/D \\ 0 & \text{for } (\widehat{f_s}/2)/D \leq f \leq \widehat{f_s}/2, \end{cases} \tag{16a}$$

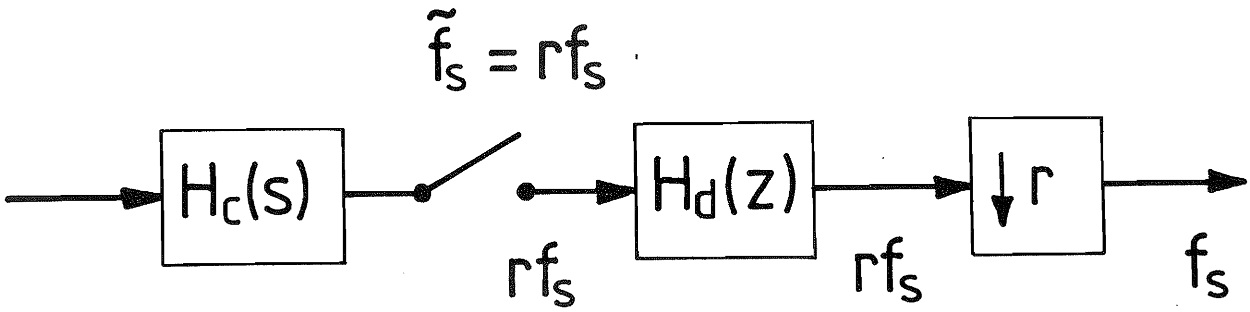
where

$$D = \max(L, M). \tag{16b}$$

- If L and M are very large integers or the sampling rate conversion factor is arbitrary, it is more beneficial to use the technique to be described in Part III of this course.

Partly Digital Anti-Aliasing Filter

• The requirements for the analog anti-imaging filter can be made significantly milder and a very linear phase performance can be achieved in the passband by using the following structure:



• In this case, the output sampling rate of the analog filter with transfer function $H_c(s)$ is $\widetilde{f}_s = rf_s$, where f_s is final sampling rate.

• This filter is followed by a decimator with transfer function $H_d(z)$ decimating by a factor of r to generate the desired output sampling rate.

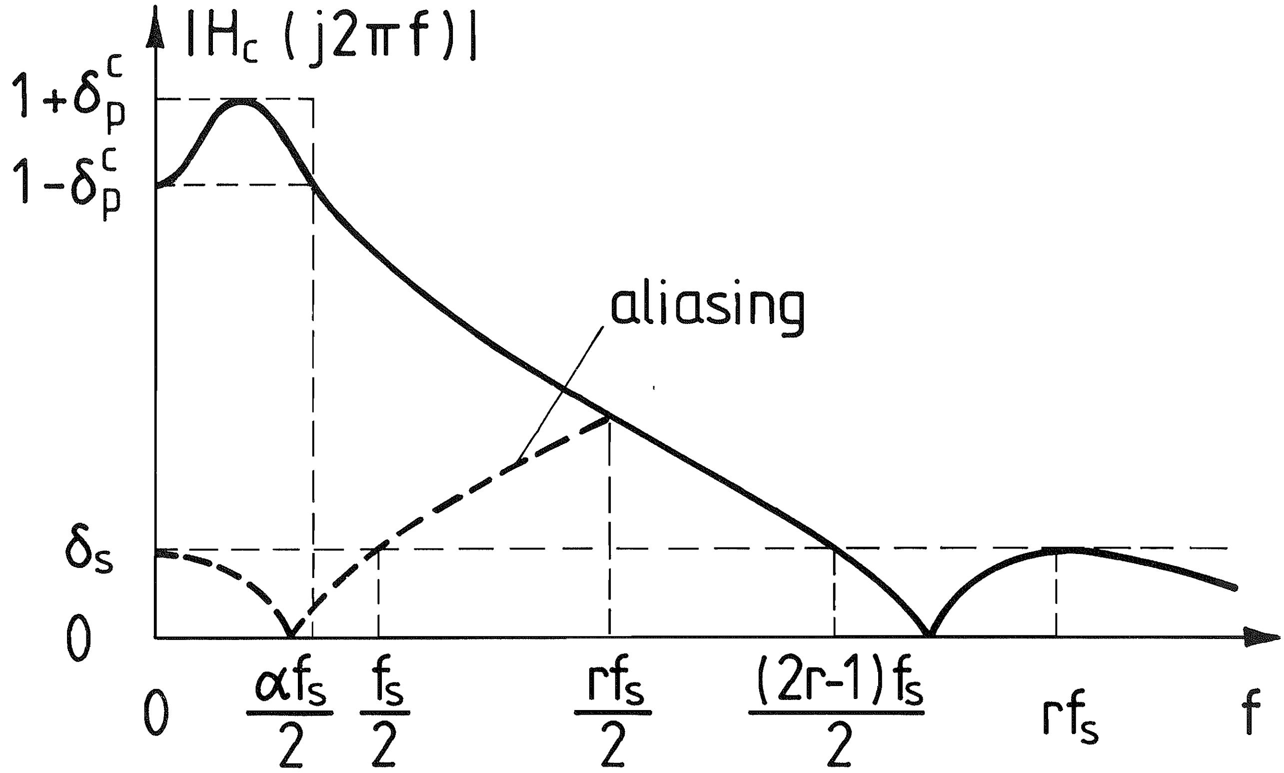
• For the overall system, the frequency response is expressible as $H_{ove}(j2\pi f) = H_c(j2\pi f)H_d(e^{j2\pi f/(rf_s)}]$.

• It is desired to design the overall system such that the maximum deviation of $|H_ove(j2\pi f)|$ from unity in the passband $[0, \alpha f_s/2]$ with $\alpha < 1$ is less than or equal to δ_p, and the maximum deviation from zero is less than or equal to δ_s for $f \geq f_s/2$.

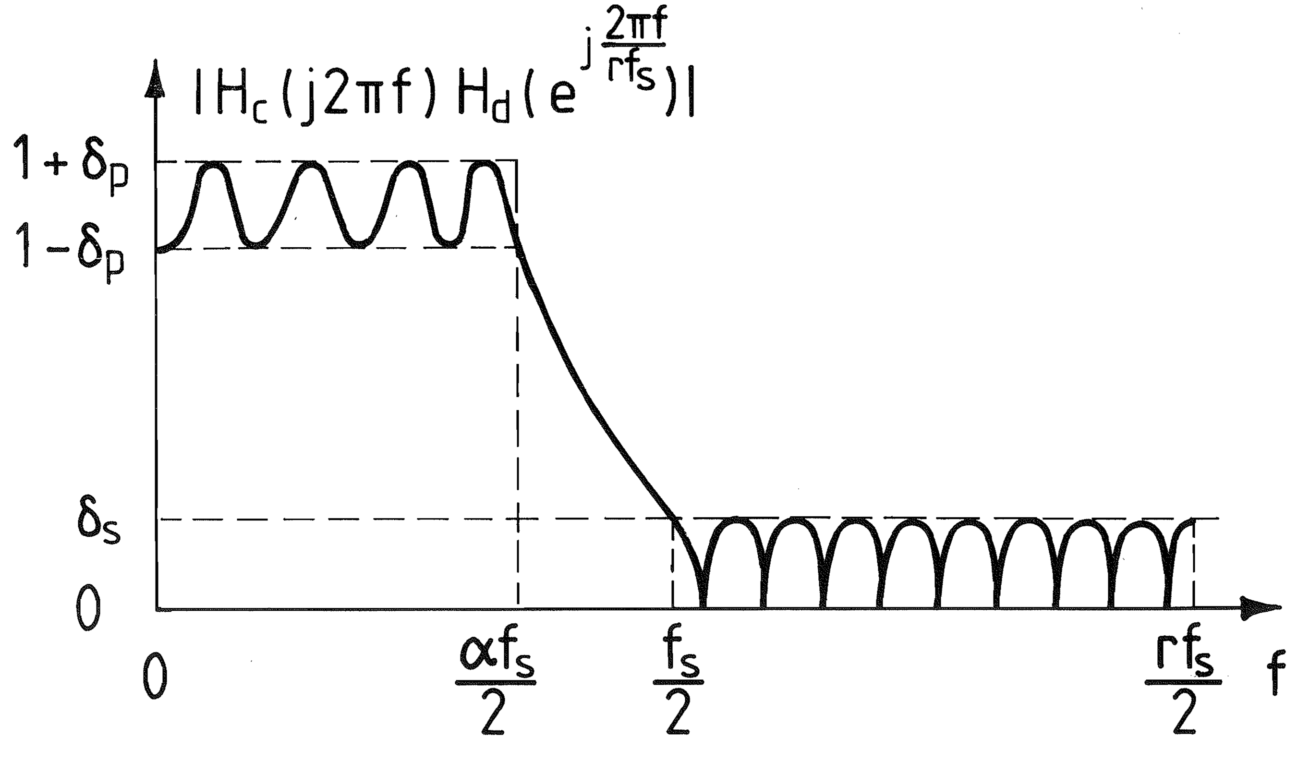
• As shown on Page 33, the design can be accomplished in two stages:

• In the first step, $H_c(s)$ is designed such that its amplitude response oscillates within $1 \pm \delta_p^c$ in the passband $[0,\ \alpha f_s/2]$, the maximum amplitude deviation from zero is less than or equal to δ_s for $f \geq (2r-1)f_s/2$, and the group delay variation around the passband average is minimized.

- Here, δ_p^c can be significantly larger than δ_p.
- When starting the stopband edge at $f = (2r-1)f_s/2$, the frequency components aliasing to the final baseband $[0,\ f_s/2]$ are well attenuated, as shown on Page 33.
- There is significant aliasing into the band $[f_s/2,\ rf_s/2]$. These components are well attenuated by the decimator.
- In the second step, $H_d(z)$ is designed such that it provides the desired performance for the overall system for $0 \leq f \leq rf_s/2$.


$|H_c(j2\pi f)|$
$1+\delta_p^c$
$1-\delta_p^c$
aliasing
δ_s
0
0
$\frac{\alpha f_s}{2}$
$\frac{f_s}{2}$
$\frac{rf_s}{2}$
$\frac{(2r-1)f_s}{2}$
rf_s
f

(a)

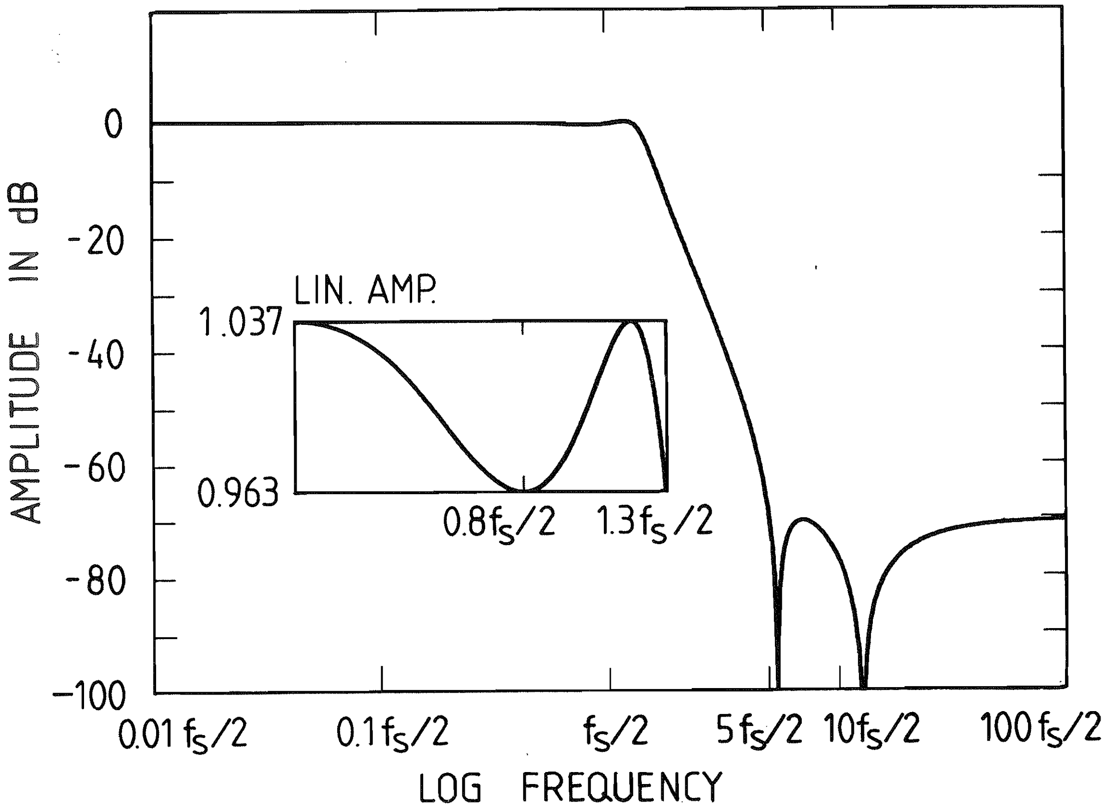

$|H_c(j2\pi f)\,H_d(e^{j\frac{2\pi f}{rf_s}})|$
$1+\delta_p$
$1-\delta_p$
δ_s
0
0
$\frac{\alpha f_s}{2}$
$\frac{f_s}{2}$
$\frac{rf_s}{2}$
f

(b)

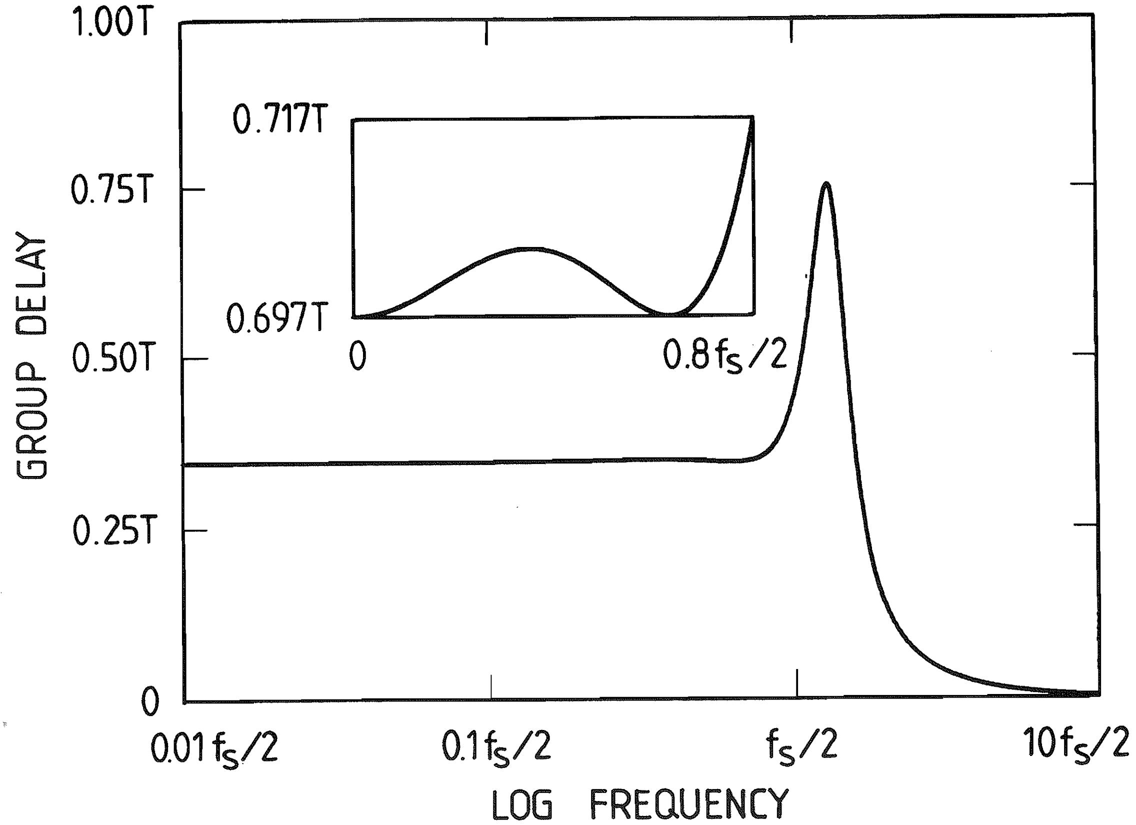
Example

- Criteria: $\alpha = 0.8$, $r = 3$, $\delta_p = 0.00576$ (0.1-dB overall passband variation), $\delta_s = 0.000316$ (70-dB stopband attenuation, suitable for a 12-bit converter), the maximum allowable group delay ripple in terms of the final sampling period T is $0.01T$.
- In the following, there are two pages illustrating the characteristics of the optimized overall design.
- Figures (a) and (b) show the amplitude and group delay responses for the optimized fourth-order analog filter. The desired group delay variation is achieved by selecting the passband ripple to be $\delta_p^c = 0.037$.
- Figure (c) shows the response of a linear-phase FIR decimator filter $H_d(z)$ required for the system to meet the amplitude criteria, as shown in Figure (d).
- If no decimator is used, then an elliptic filter of order nine is required to meet the same criteria. The phase response of this filter is very nonlinear in the passband. Furthermore, the tuning of the elliptic filter is significantly more difficult.

Example Partly Digital Anti-Aliasing Filter

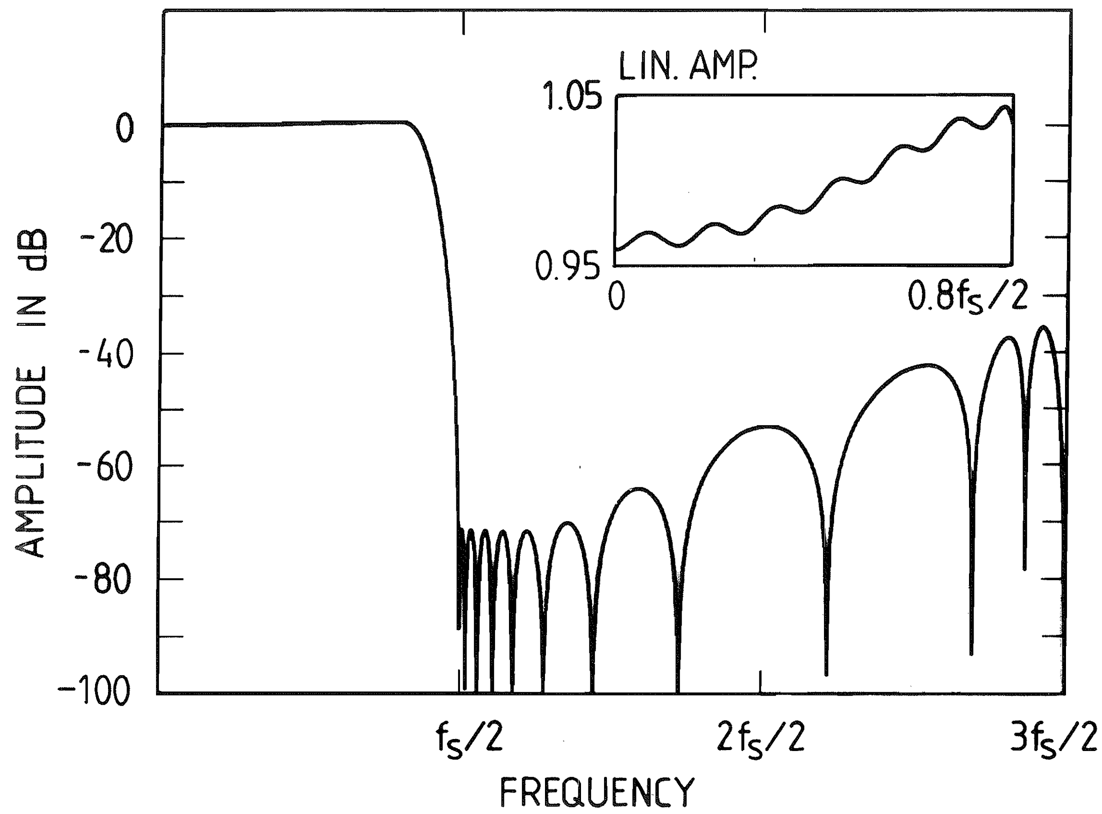


(a)

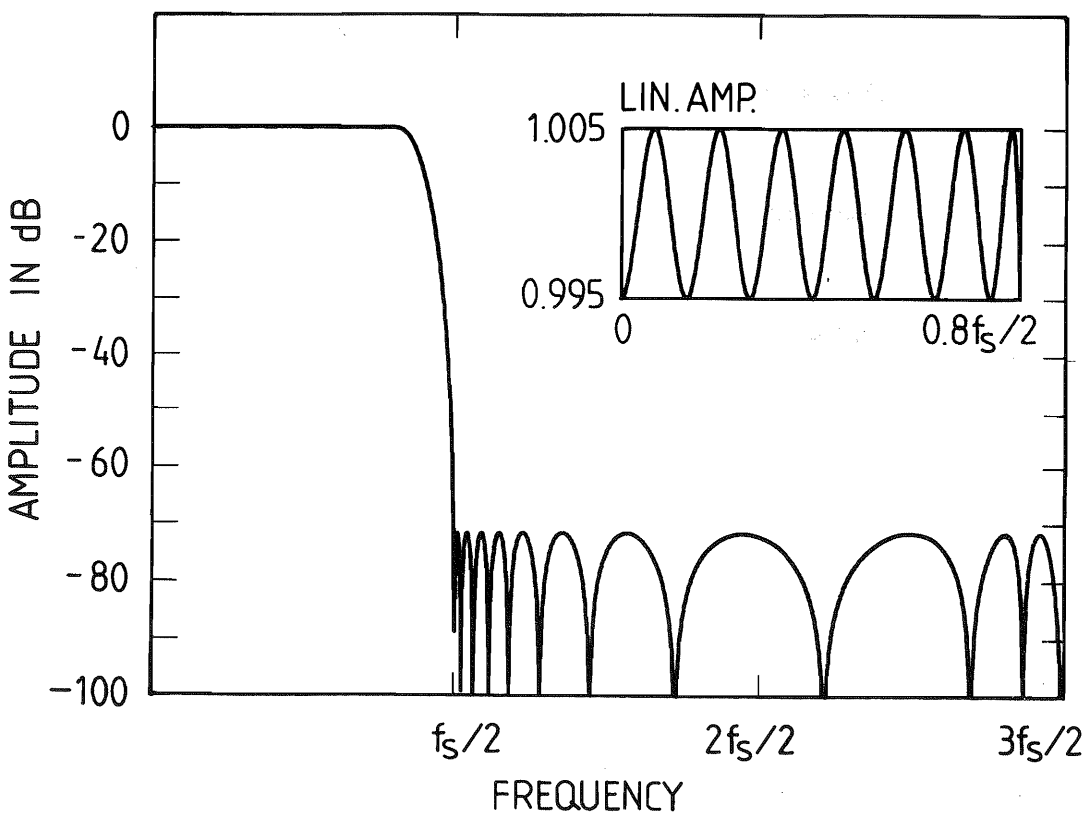


(b)

Example Partly Digital Anti-Aliasing Filter



(c)



(d)

Sigma-Delta A/D Conveter

- An extreme case is an A/D conveter based on the use of sigma delta modulator.
- In the case of a converter shown on the next page, $r = 64$ and a simple analog RC filter can be used.

A Stereo Sigma-Delta A/D-Converter with Oversampling Ratio of 64

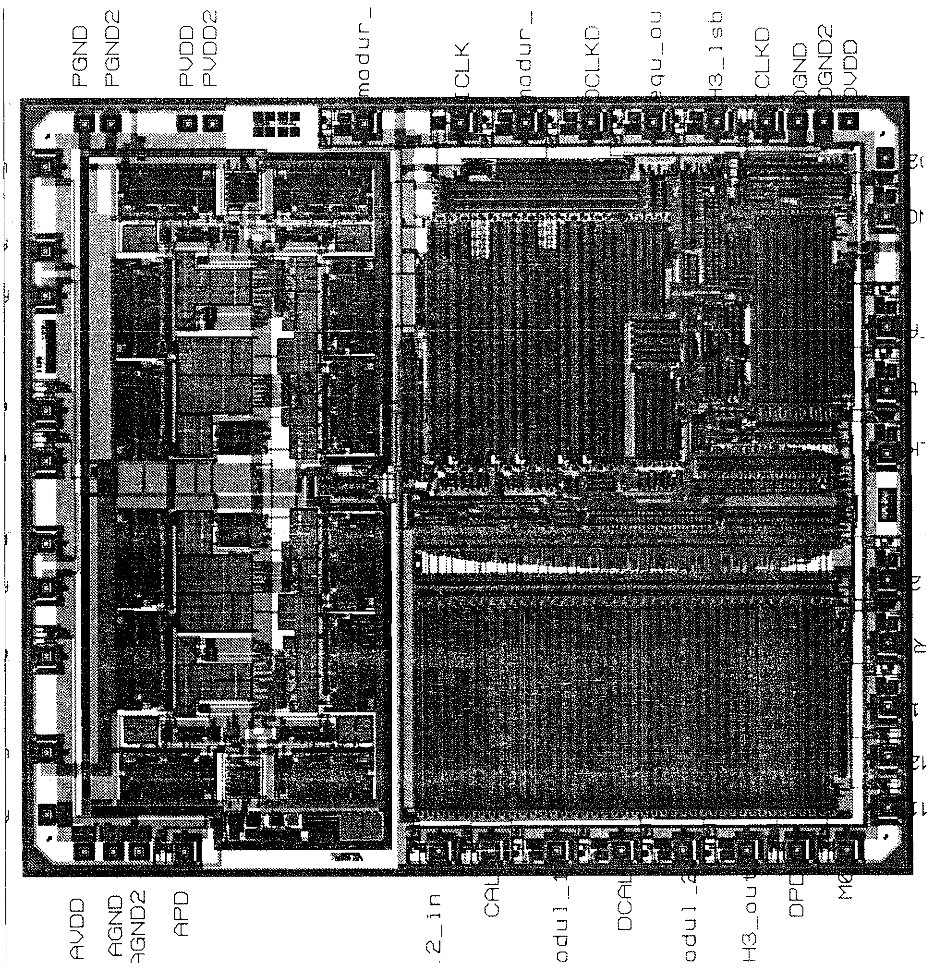
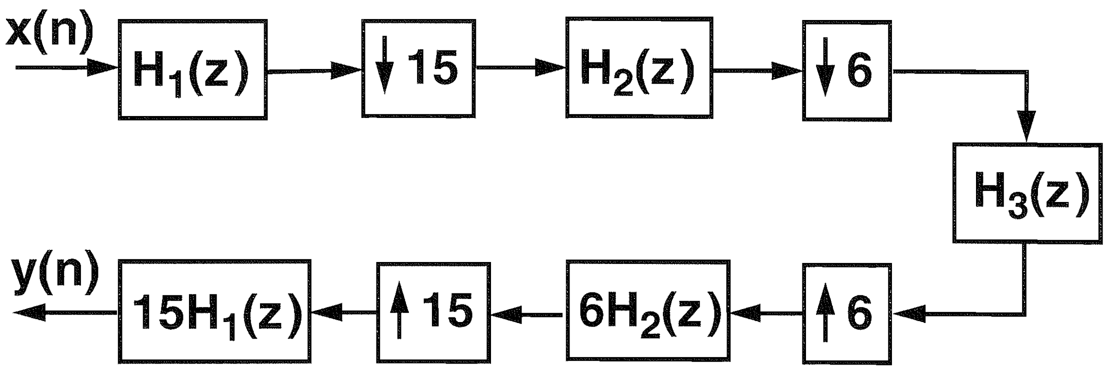


Figure 1: AD-converter for audio use

Example: Passband edge = $0.0095 \cdot f_S/2$
Stopband edge = $0.01 \cdot f_S/2$
Passband ripple = 0.001
Stopband ripple = 0.0001 (80 dB)

Direct-form conventional FIR filter of order 15590:
7796 multiplications per input sample

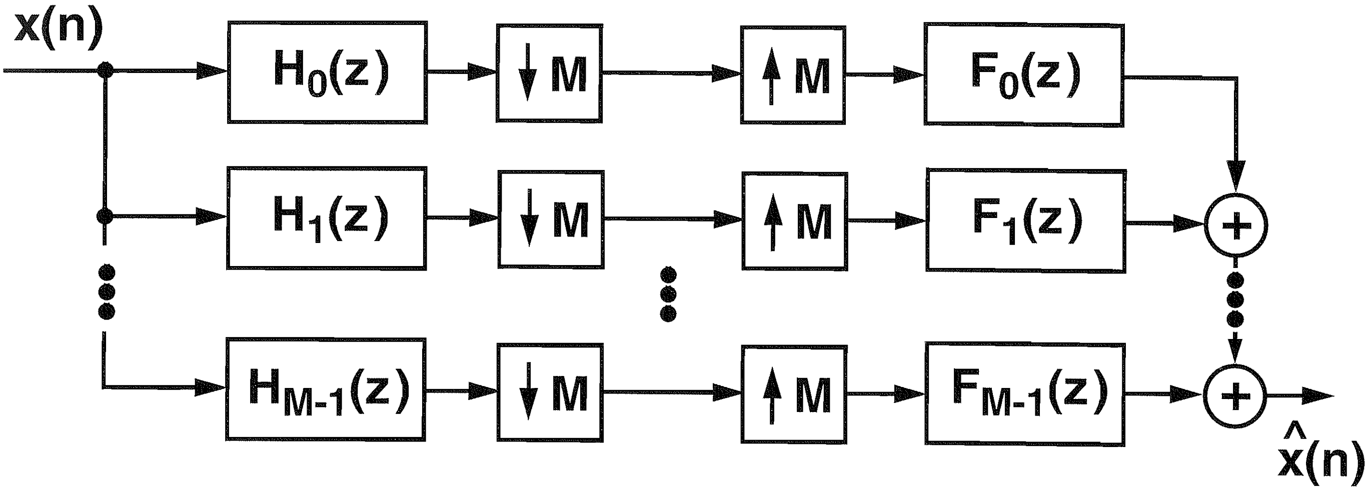
FIR filter implemented using decimation and interpolation:



$H_1(z)$: order = 39
$H_2(z)$: order = 40
$H_3(z)$: order = 197

Only 4.16 multiplications per input sample

A Filter Bank for Subband Coding



The $H_k(z)$'s and $F_k(z)$'s can be designed such that $\hat{x}(n)=x(n-K)$. This filter bank is used for subband coding.