HOMEWORK 1: Design of linear-phase FIR filters

Consider the following specifications for the zero-phase frequency response of a linear-phase FIR filter:

\[
0.99 \leq H(\omega) \leq 1.01 \quad \text{for} \quad 0 \leq \omega \leq 0.4\pi \\
-0.01 \leq H(\omega) \leq 0.01 \quad \text{for} \quad 0.6\pi \leq \omega \leq \pi.
\]

Design a filter with the minimum order to meet these criteria both by (a) the windowing and by (b) the Remez-algorithm (MPR algorithm). In both cases, evaluate the zero-phase frequency response and check whether it meets the given criteria. What are the minimum filter orders? What is characteristics of the impulse responses? What was your window function?

Consider the following specifications for the zero-phase frequency response of a linear-phase FIR filter:

\[
-0.01 \leq H(\omega) \leq 0.01 \quad \text{for} \quad 0 \leq \omega \leq 0.1\pi \\
0.9 \leq H(\omega) \leq 1.1 \quad \text{for} \quad 0.2\pi \leq \omega \leq 0.4\pi \\
-0.01 \leq H(\omega) \leq 0.01 \quad \text{for} \quad 0.7\pi \leq \omega \leq \pi.
\]
Design of a filter with the minimum order using the Remez algorithm such that in the transition bands the amplitude response is forced to be less than or equal to 1.1. Evaluate the resulting response and make sure that it meets the given requirements.

**What to return?** Matlab commands required to designing the above filters (command ‘diary’ may help, useless iterations can be edited away), the resulting impulse responses coefficients both in numbers and in graphical forms, and answers to the above-mentioned questions.
Consider the following specifications for IIR filters:

\[-1 \leq 20 \log_{10} |H(e^{j\omega})| \leq 0 \quad \text{for} \quad 0 \leq \omega \leq 0.4\pi\]
\[-60 \leq 20 \log_{10} |H(e^{j\omega})| \quad \text{for} \quad 0.6\pi \leq \omega \leq \pi,\]

that is, the maximum allowable passband variation is 1 dB, and the minimum stopband attenuation is 60 dB. Design a Butterworth, a Chebyshev Type I (Chebyshev), a Chebyshev Type I (inverse Chebyshev), and an elliptic filter to meet these criteria with minimum orders. What is the minimum order in each case? Evaluate the frequency responses and study their performance.

In Matlab, you can use the function ‘yulewalk’ to design arbitrary-magnitude IIR filters. Design a ninth-order IIR filter to approximate the following desired response:

\[|H(e^{j\omega})| = \begin{cases} 
(\omega/0.4\pi)^2, & 0 \leq \omega \leq 0.4\pi \\
0, & 0.6\pi \leq \omega \leq \pi.
\end{cases}\]

Evaluate the frequency response and study the performances of the amplitude and phase responses. Does it correspond to the desired one?
What to return? Matlab commands required to designing the above filters, the coefficients of the transfer functions as well as the answers to the above-mentioned question. Also, the filter frequency responses.

HOMEWORK 3: Design of a linear-phase FIR filter as a cascade of a periodic and nonperiodic filter

Consider the specifications: $\omega_p = 0.0375\pi$, $\omega_s = 0.075\pi$, $\delta_p = 0.01$, $\delta_s = 0.001$. Synthesize the transfer function in the form $H(z) = F(z^L)G(z)$ using the Remez algorithm in a manner as described in the lecture notes. Determine $L$ in such a way that the overall number of multipliers is minimized.

What to return? Matlab commands, the responses and coefficients of the subfilters, and the response of the overall filter.