PART III: Design of various kinds of digital filters meeting the same criteria

- This part shows how the same filter criteria can be met by various kinds of digital filters.
- Futhermore, it is shown that after proper reasoning we are able to end up filters with a drastically reduced complexity.

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FISH RESEARCH

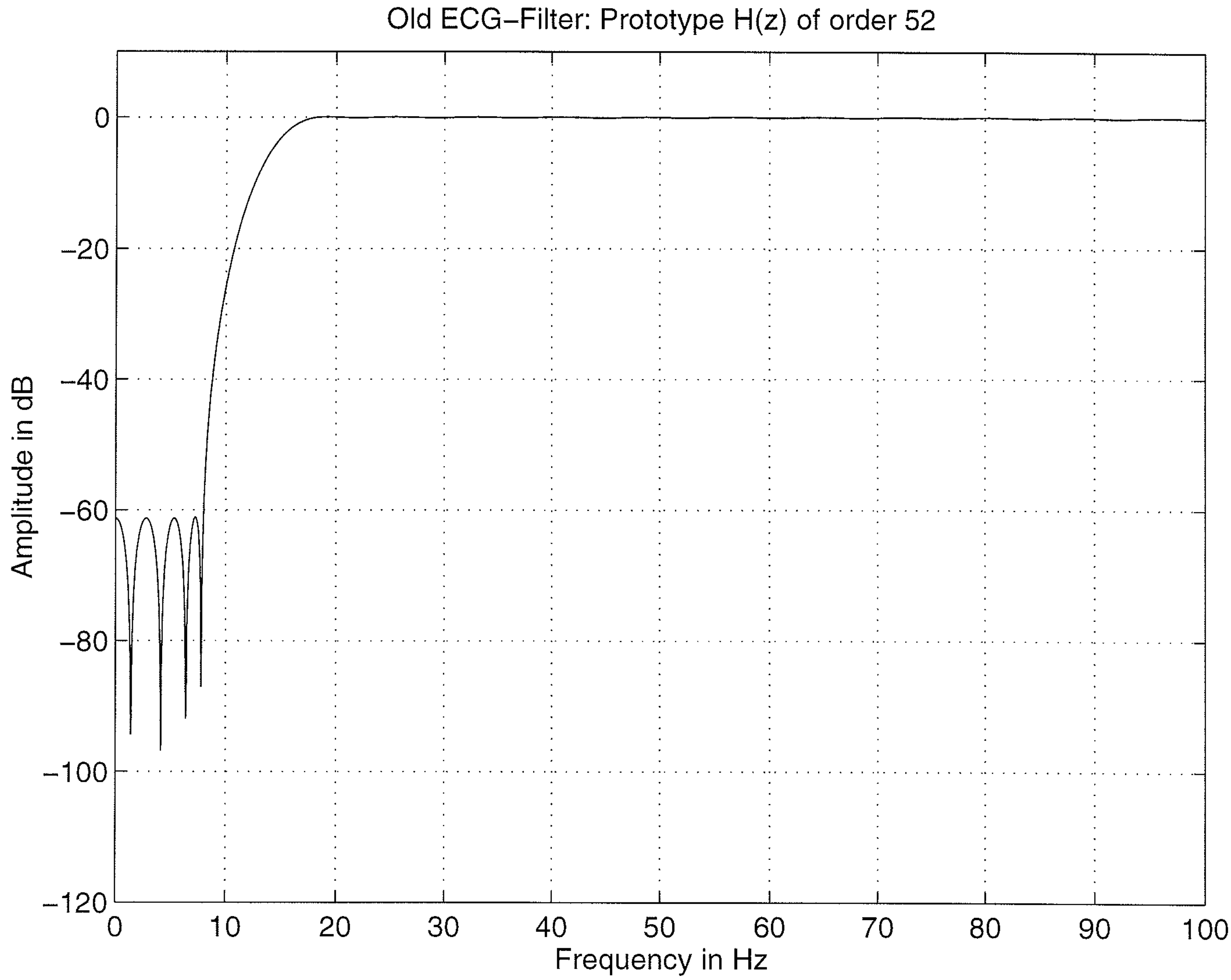
- In the end of this pile of lecture notes there is an article which considers the use of the brown trout to test the quality of the surface raw water.
- In this article, several filters have been designed.
- Here, we concentrate on the design of the ECG-filter and the total activity filter.
- Matlab-files, called fishi1.m and fishi2.m, used for designing these filters can also be found in the end of these lecture notes.

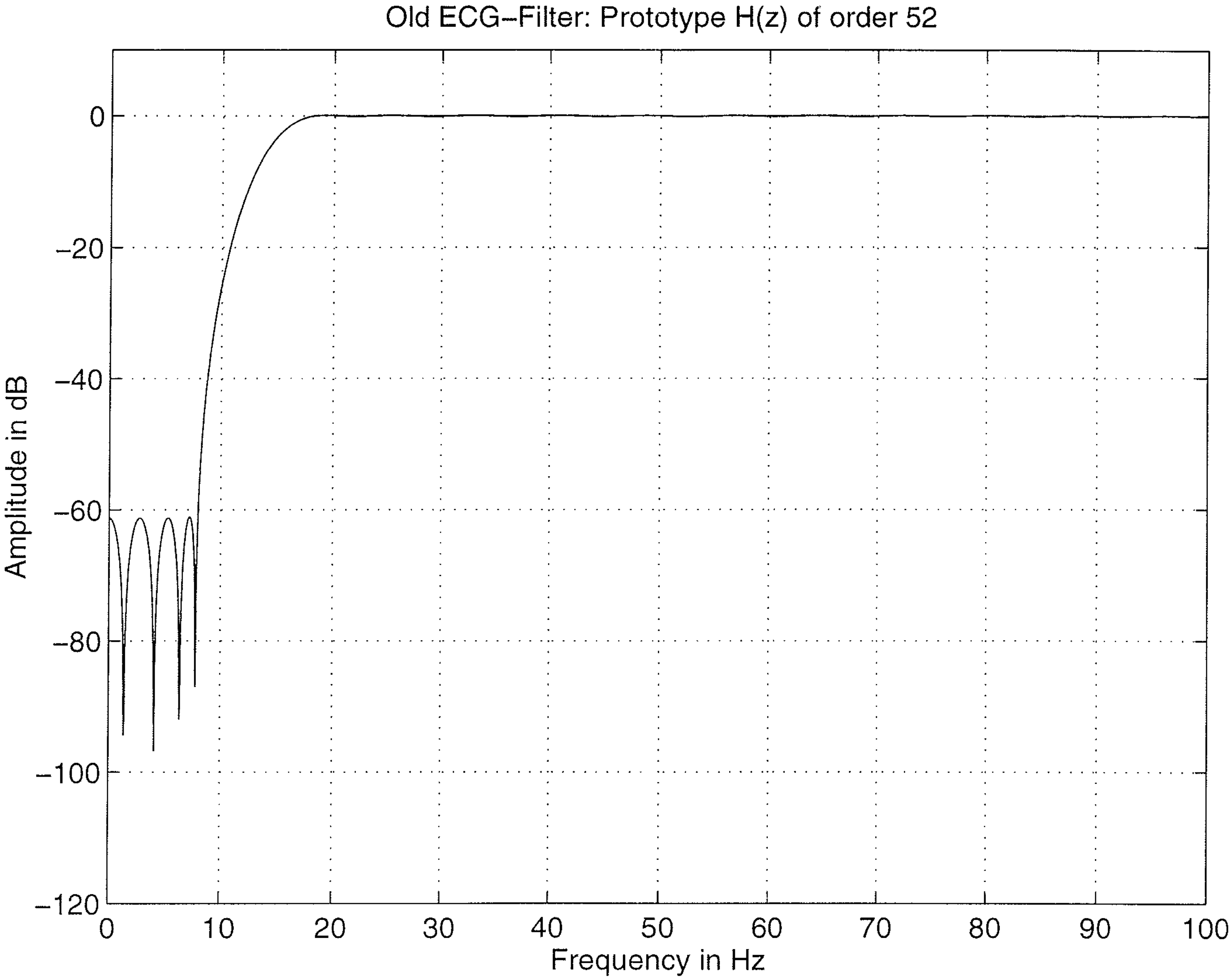
ECG FILTER: OLD DESIGN

- It is desired to design for $f_s = 200$ Hz a filter having the stopband regions from 0 Hz to 2 Hz, from 48 Hz to 52 Hz, and from 98 Hz to 100Hz. The passband regions are from 4.5 Hz to 45.5 Hz and from 54.5 Hz to 95.5 Hz. The passband ripple is $\delta_p = 0.01$ and the stopband ripple is $\delta_s = 0.001$.
- In the article in the end of this pile, they first design a prototype highpass filter $H(z)$ for $f_s = 200$ Hz in such a way that the stopband region is from 0 Hz to $4 \cdot 2 = 8$ Hz and the passband region is from $4 \cdot 4.5 = 18$ Hz to 100 Hz. In the ω-scale, the stopband edge is 0.08π and the passband edge is 0.18π. Using the Remez algorithm (see Matlab-file fishi1.m), the minimum filter order is 52.
- Because of the periodicity of the overall specifications, the desired overall filter is then automatically $H(z^4)$, that is, a transfer function obtainable from $H(z)$ by using instead of a unit delay z^{-1} a delay block z^{-4}. The implementation of this filter is very

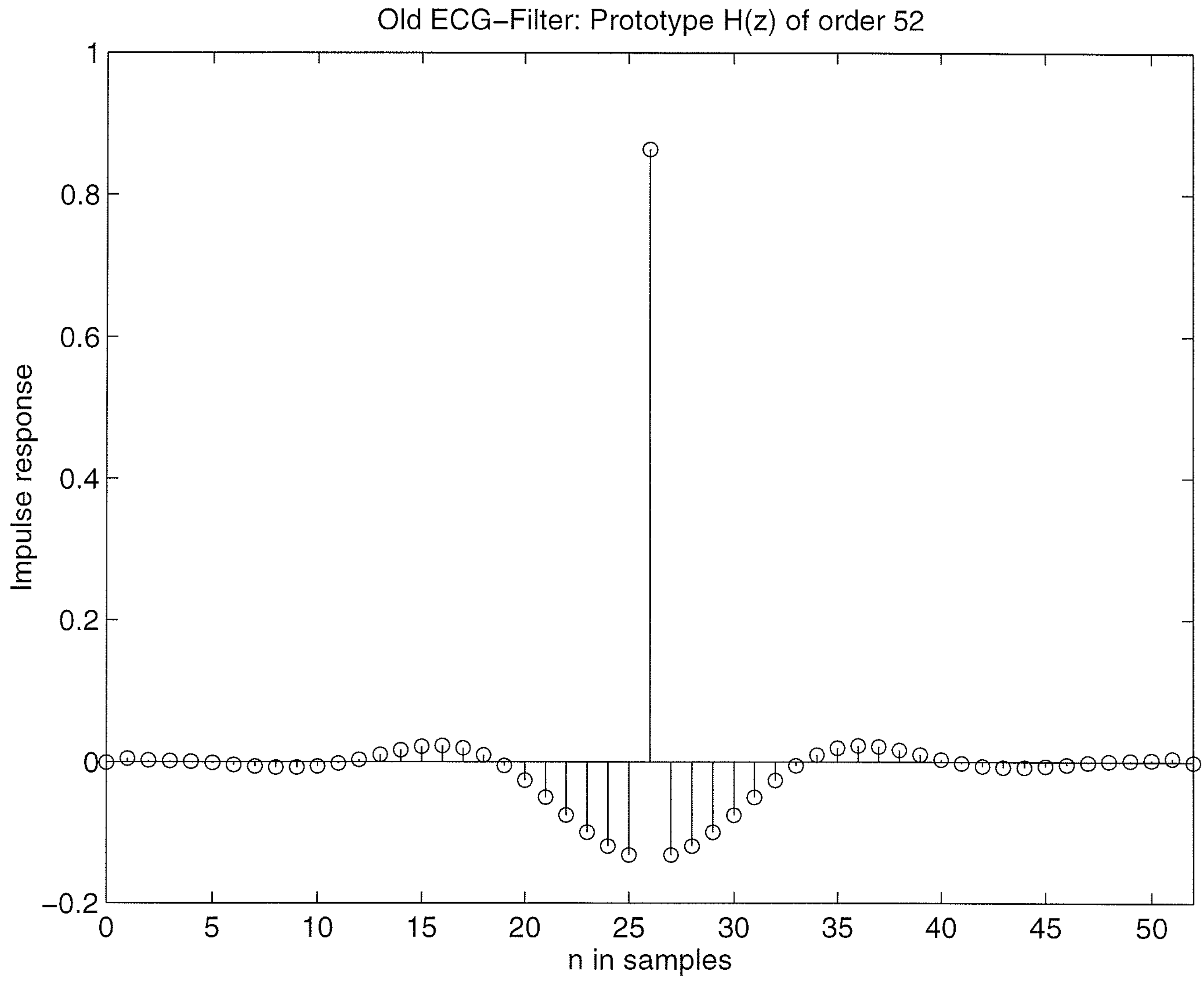
straightforward with the aid of some signal processors, where during the same instruction cycle it is possible to delay the signal by four unit samples instead of one unit sample.

- Note that by replacing z^{-1} by z^{-4} the frequency axis is shrunk by by a factor of four and 400 Hz is mapped to 100 Hz so that there are several periods of the prototype filter in the frequency band of interest, that is, in the region from 0 Hz to 100 Hz.
- The number of multipliers required by this design is approximately one fourth compared to the conventional direct-form FIR filter whose order is approximately $4 \cdot 52 = 208$. Therefore, the code length is drastically reduced.
- In the following there are four transparencies illustrating the performance of this design.

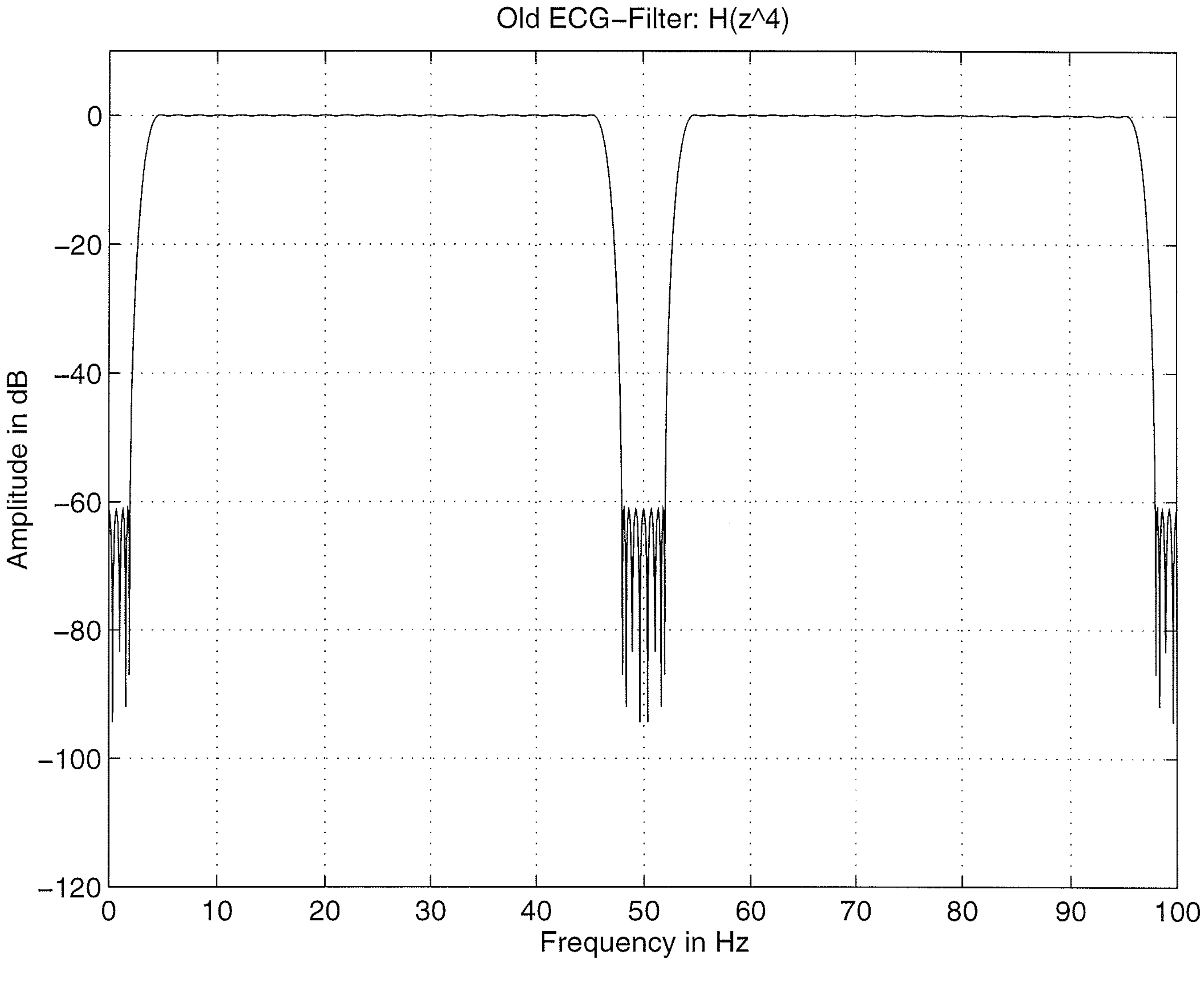

Old ECG−Filter: Prototype H(z) of order 52
0
−20
−40
−60
−80
−100
−120
Amplitude in dB
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

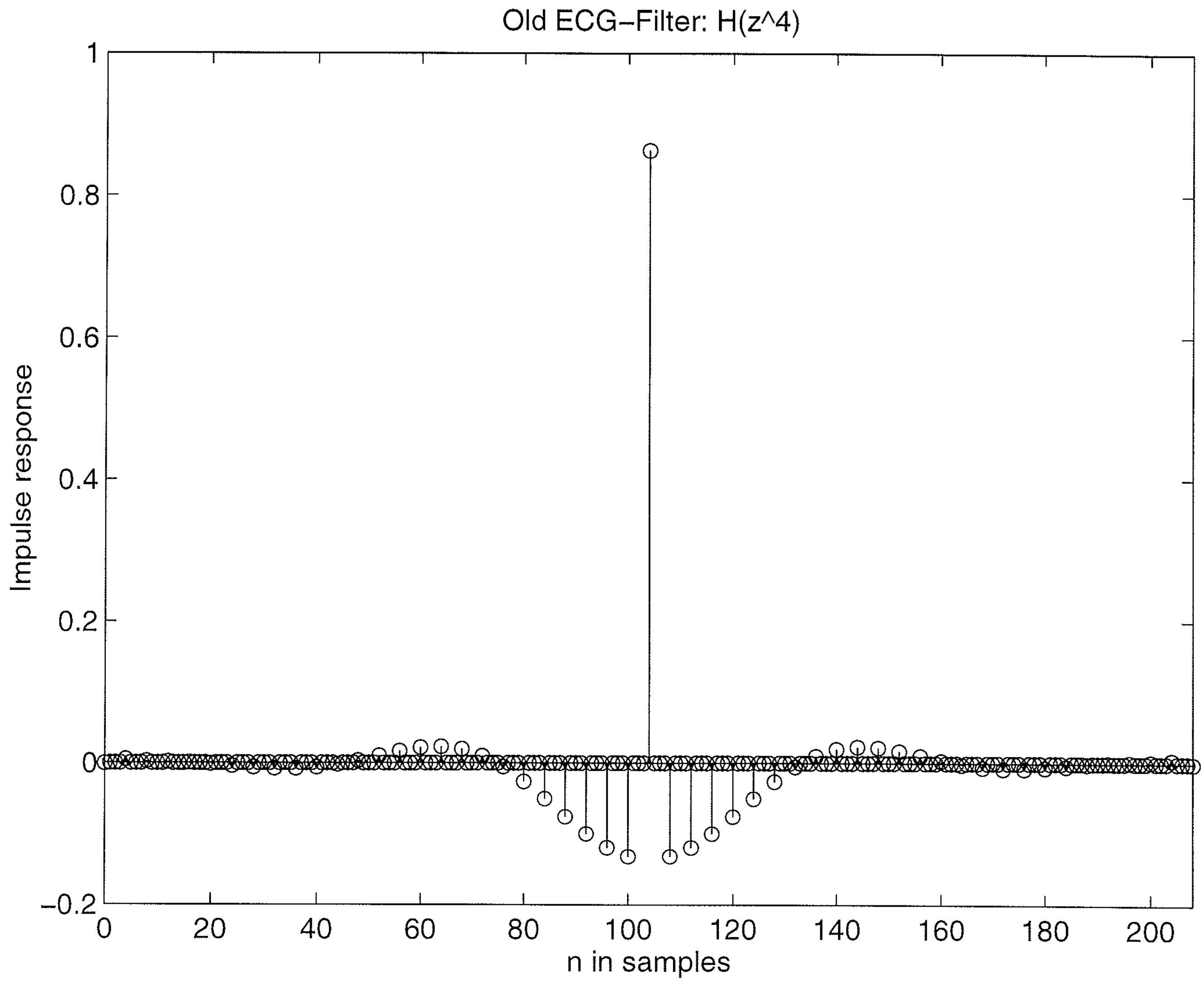


Old ECG–Filter: Prototype H(z) of order 52
Amplitude in dB
Frequency in Hz
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100



Old ECG−Filter: Prototype H(z) of order 52
1
0.8
0.6
0.4
0.2
0
−0.2
Impulse response
0
5
10
15
20
25
30
35
40
45
50
n in samples


Old ECG−Filter: H(z^4)
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz


Old ECG−Filter: H(z^4)
Impulse response
n in samples
1
0.8
0.6
0.4
0.2
0
−0.2
0
20
40
60
80
100
120
140
160
180
200

ECG FILTER: NEW DESIGN

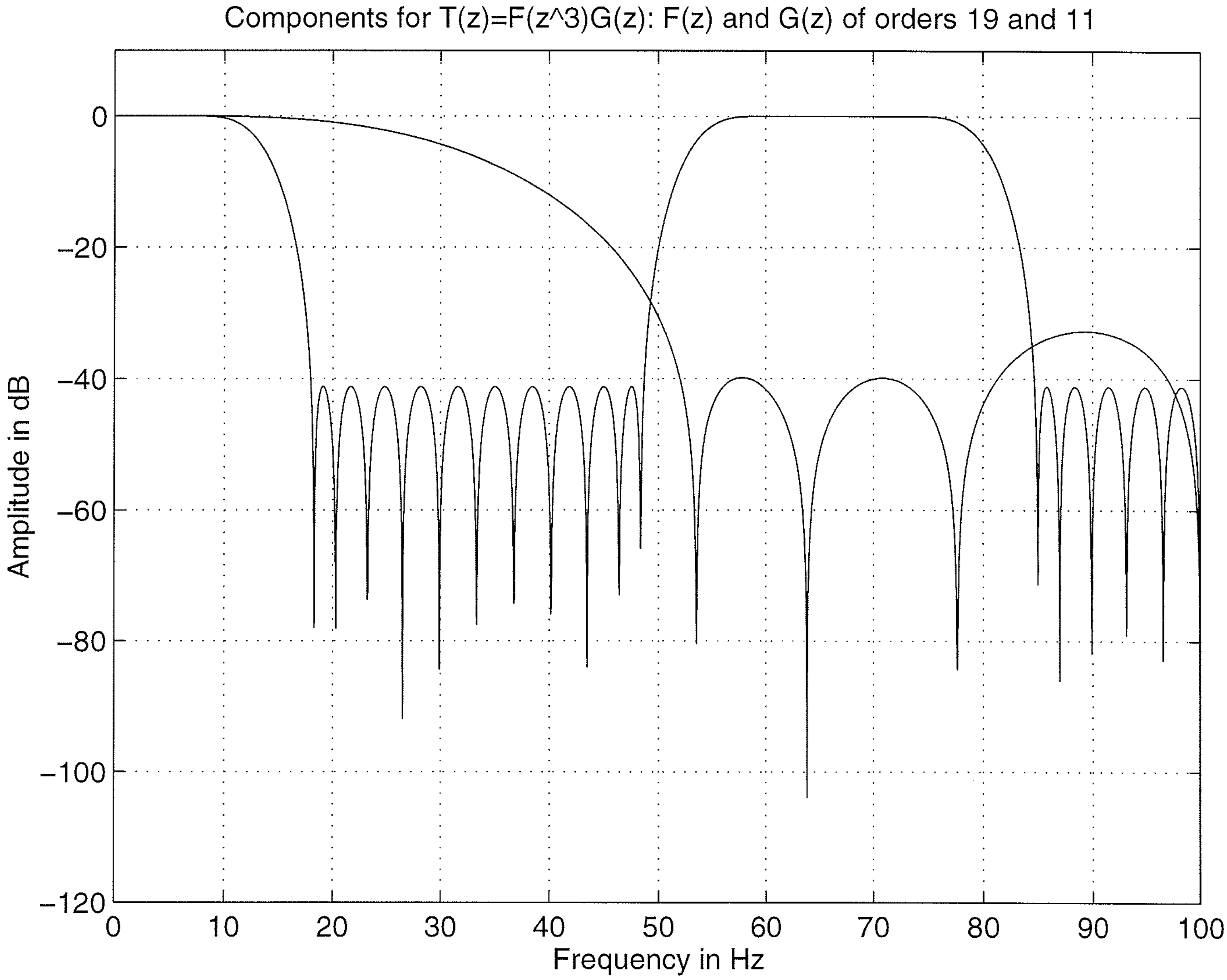
- Here we design the protype filter in the form $H(z) = z^{-M} - T(z)$, $T(z) = F(z^3)G(z)$, where M is half the order of $T(z)$. The complementary filter $T(z)$ is a lowpass filter with the passband and stopband edges in the ω-scale at $\omega_p = 0.08\pi$ and $\omega_s = 0.18\pi$. The passband and stopband ripples are $\delta_p = 0.001$ and $\delta_s = 0.01$.

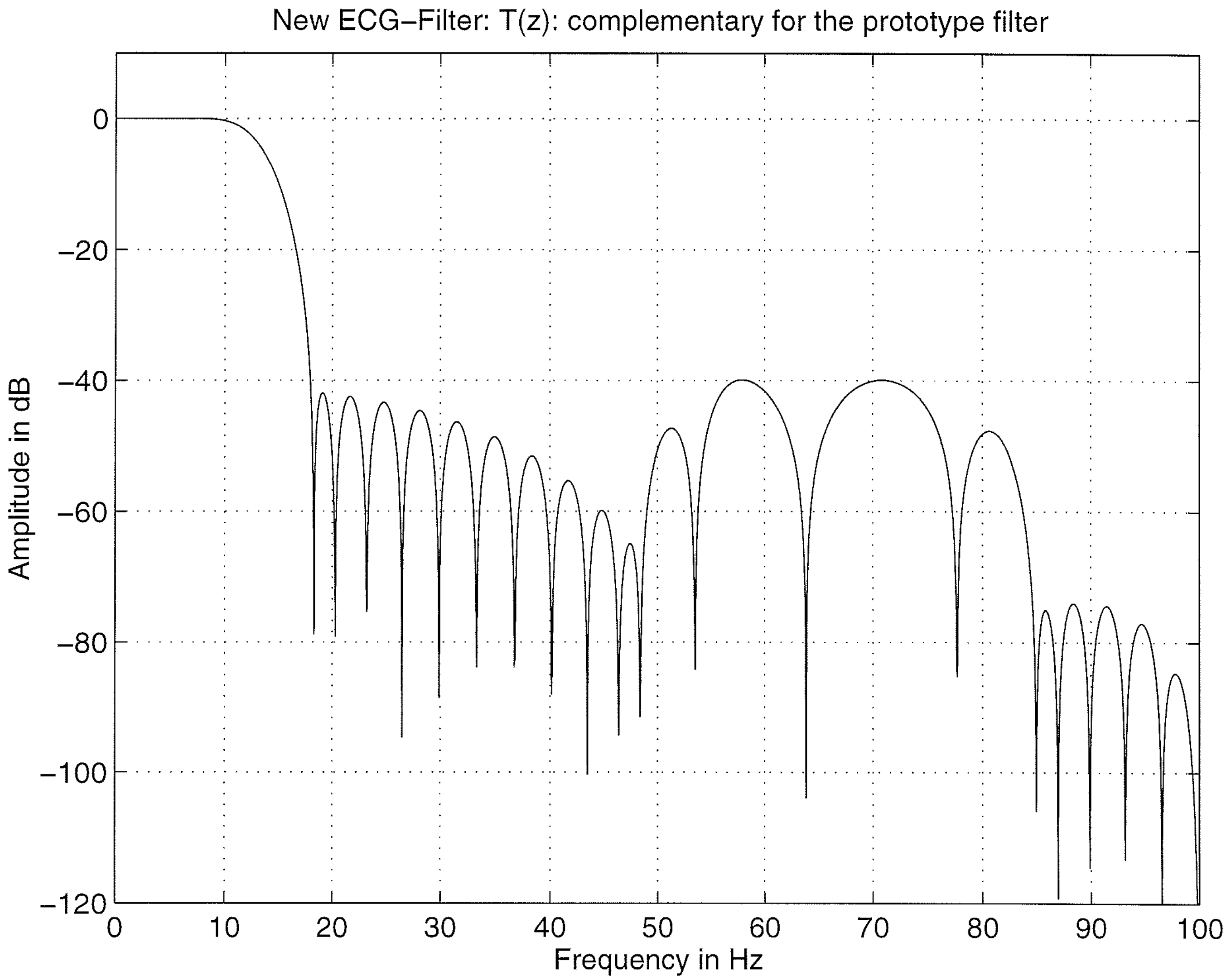
- The desired overall filter $T(z)$ is obtained by designing $F(z)$ in such a way that its passband and stopband edges are located at $3\omega_p$ and $3\omega_s$. The passband and stopband ripples are $\delta_p/2$ and δ_s. For $G(z)$ the ripples are the same. The passband region is $[0, \omega_p]$, whereas the stopband region is $[\omega_{s1}, \omega_{s2}]$ with $\omega_{s1}, \omega_{s2} = 2\pi/3 \pm (2\omega_s + \omega_p)/3$.

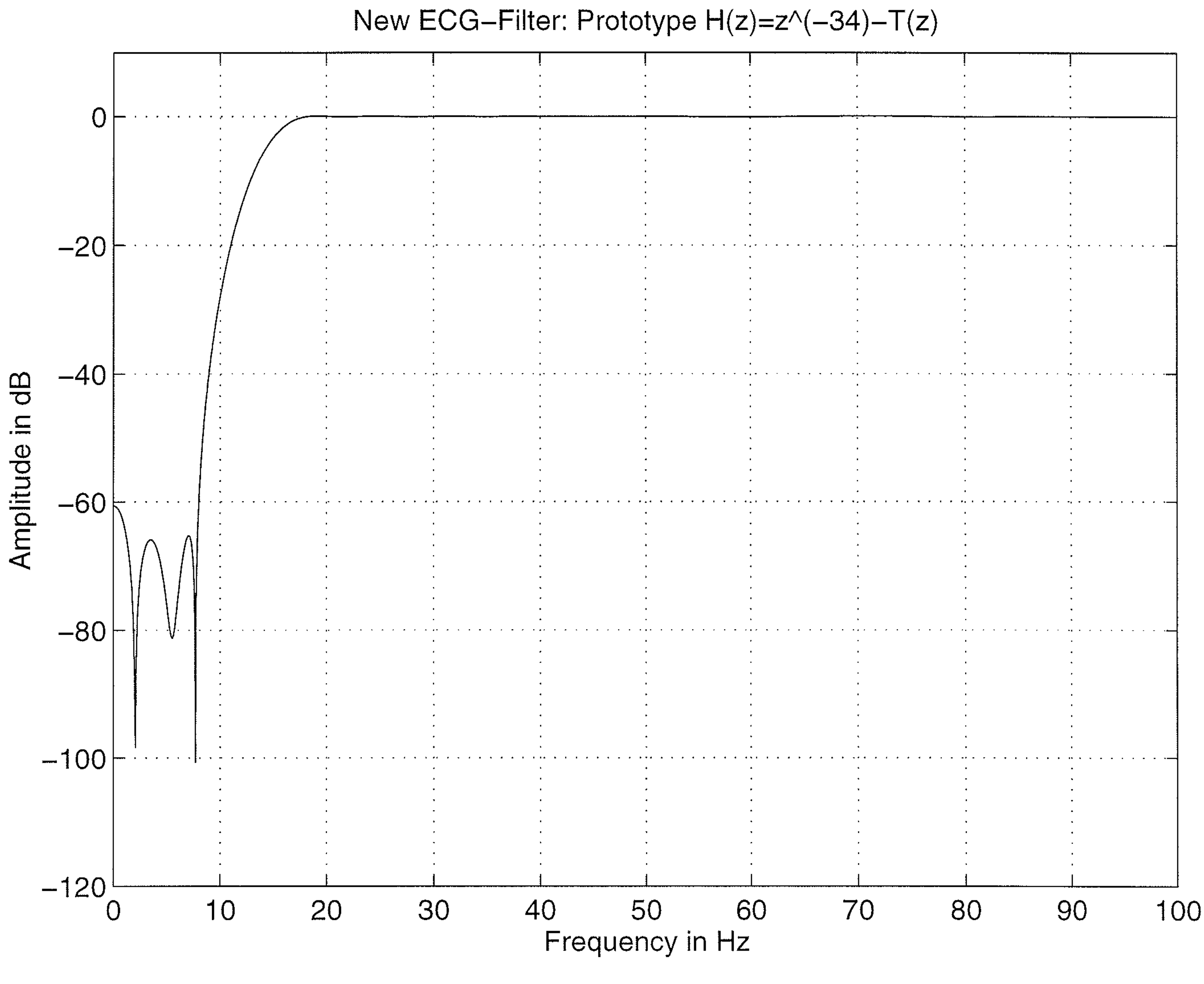
- The given criteria are met by $F(z)$ of order 19 and $G(z)$ of order 11. The overall order of $T(z)$ is thus $3 \cdot 19 + 11 = 68$ so that $M = 34$.

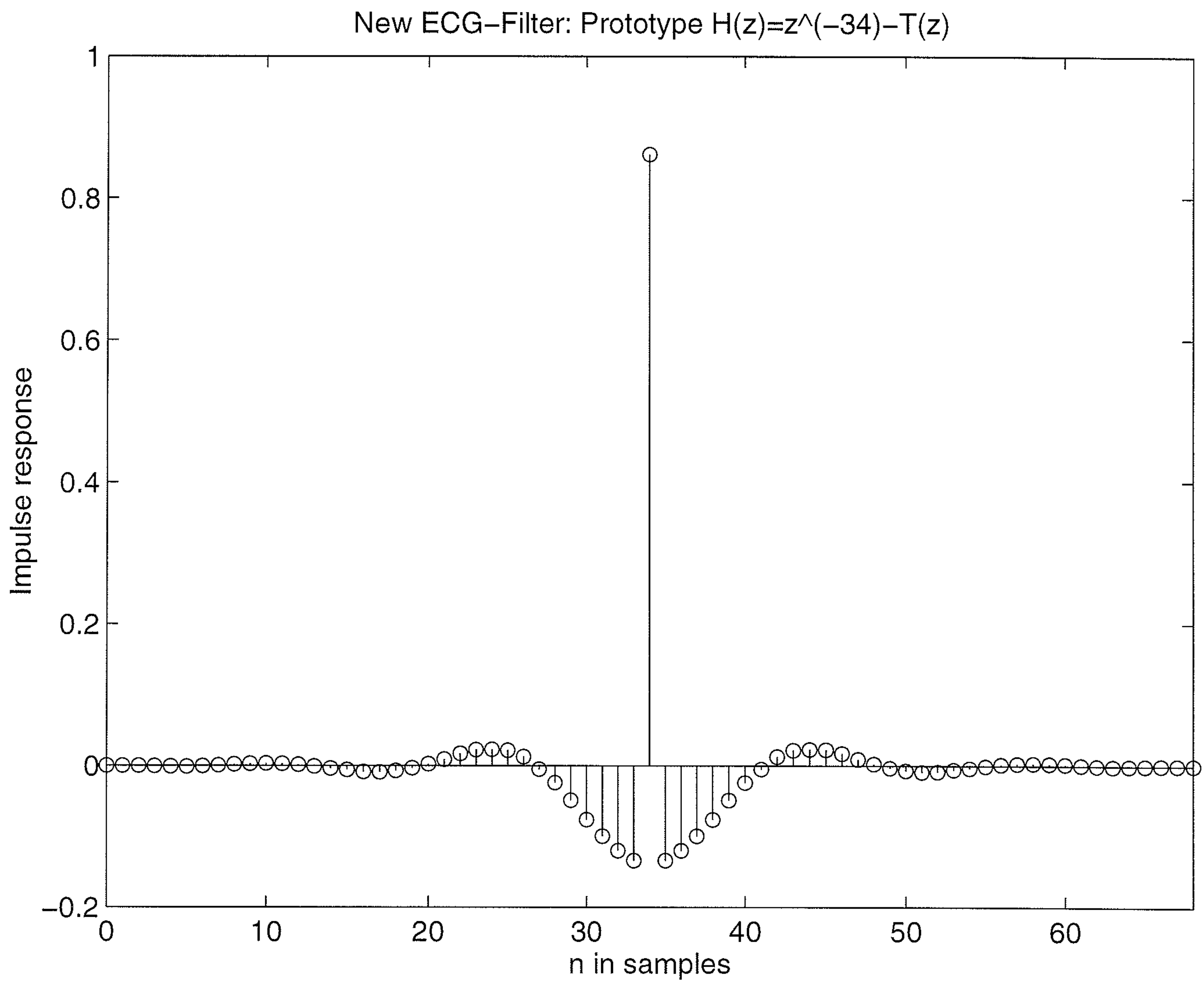
- The prototype filter is thus $H(z) = z^{-34} - F(z^3)G(z)$ and the overall filter $H(z^4) = z^{-136} - F(z^{12})G(z^4)$.

- In practical implementation using a signal processor, the delay z^{-136} can be shared with $F(z^{12})$.
- In normal signal processor implementations, the coefficient symmetry is not worth exploiting. The old design requires $52 + 1 = 53$ multipliers, whereas the new design requires $19 + 1 + 11 + 1 = 32$ multipliers at the expense of the increase in the overall filter order (from 208 to 272).
- In the following there are six transparencies illustrating the performance of this design.

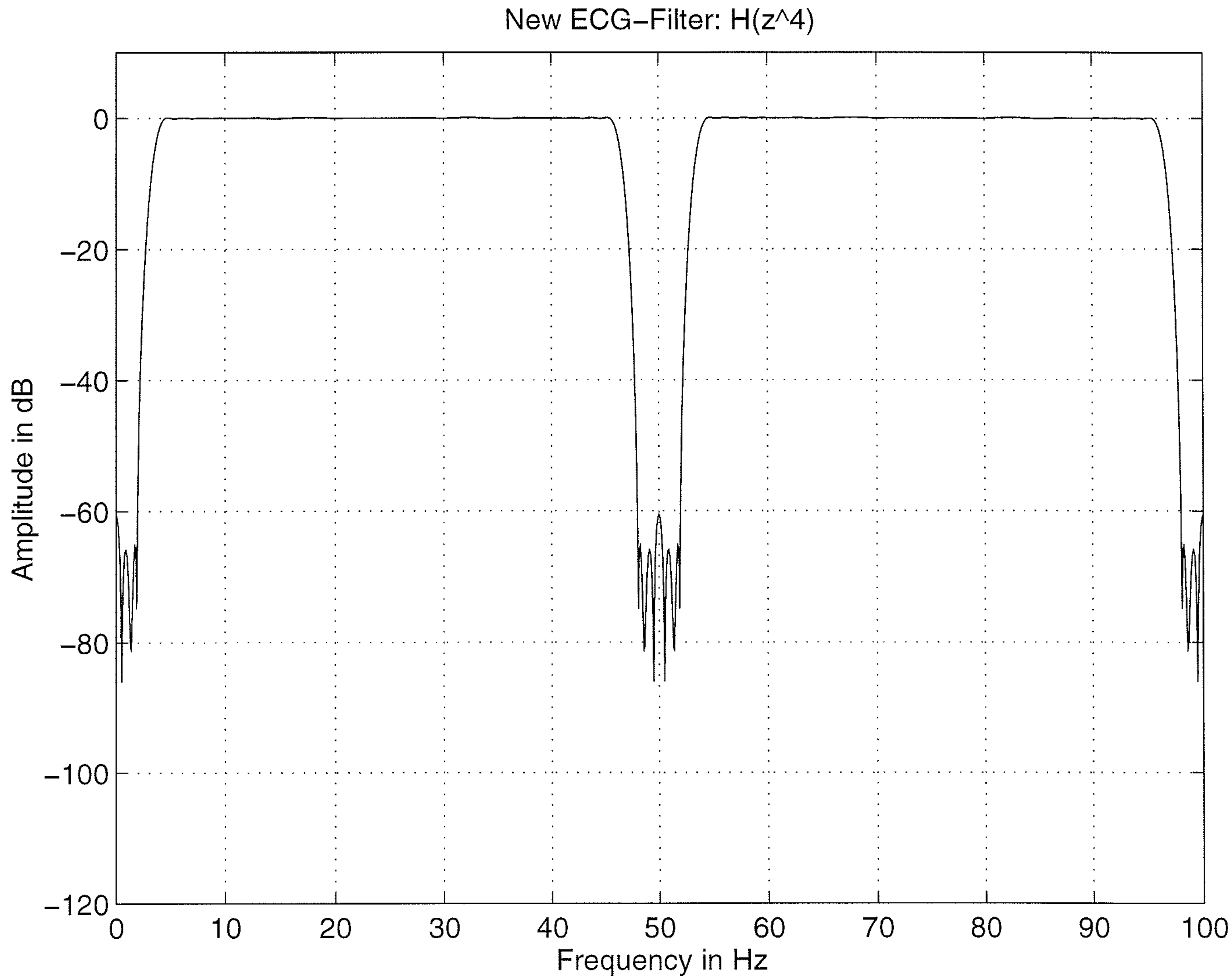

Components for T(z)=F(z^3)G(z): F(z) and G(z) of orders 19 and 11
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

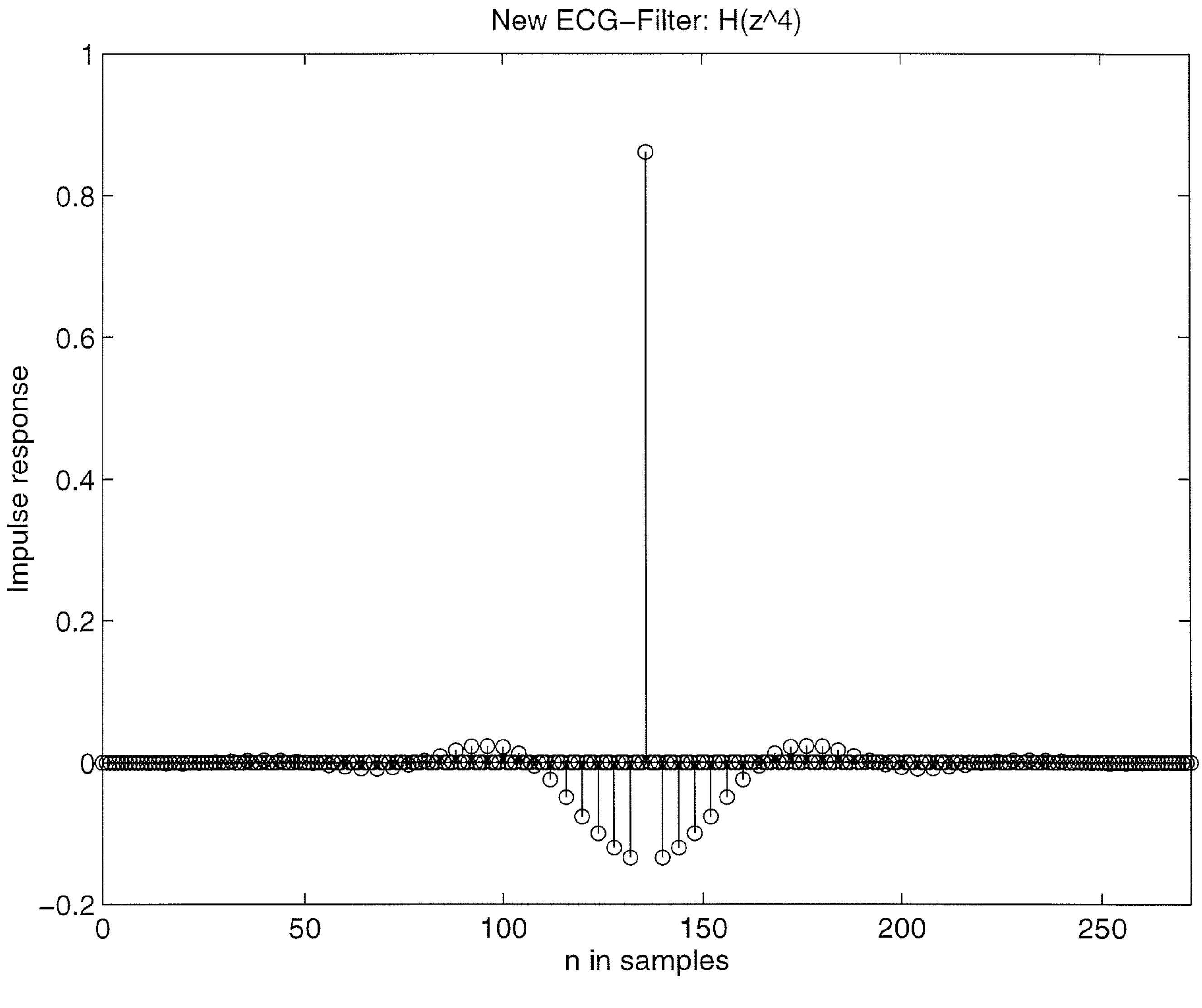

New ECG–Filter: T(z): complementary for the prototype filter
Amplitude in dB
0
–20
–40
–60
–80
–100
–120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz


New ECG−Filter: Prototype H(z)=z^(−34)−T(z)
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz



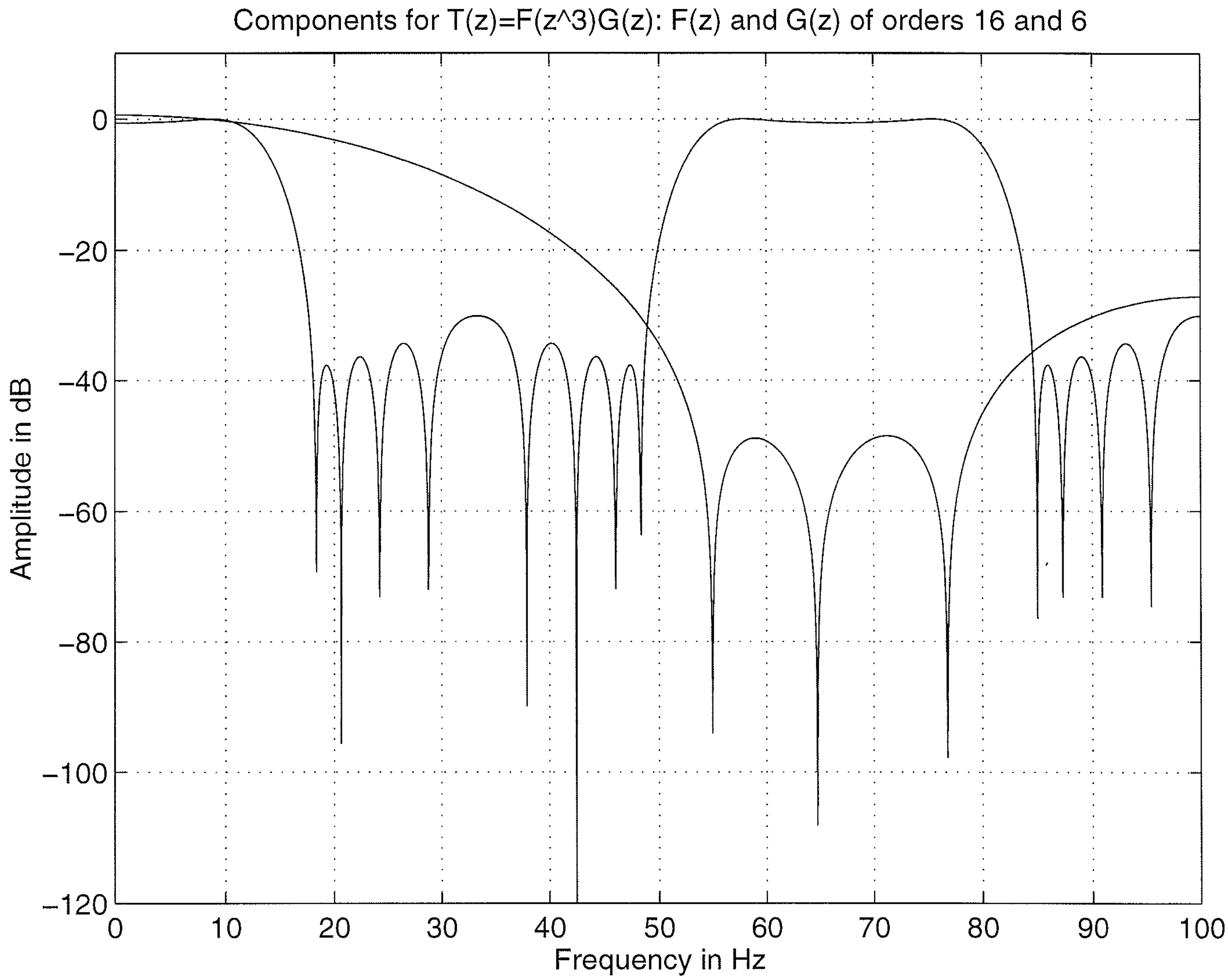
New ECG−Filter: Prototype H(z)=z^(−34)−T(z)
Impulse response
n in samples
1
0.8
0.6
0.4
0.2
0
−0.2
0
10
20
30
40
50
60

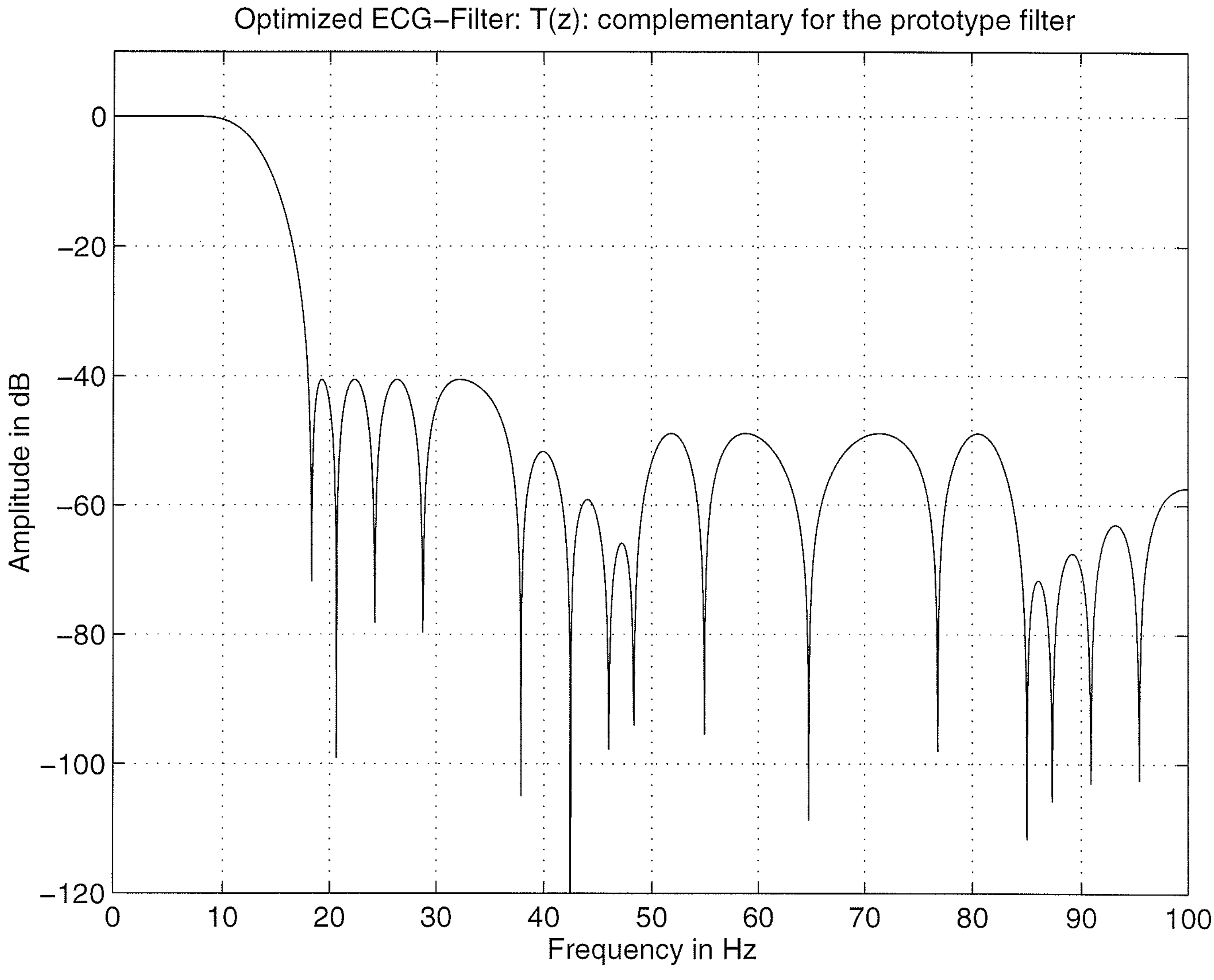

New ECG−Filter: H(z^4)
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

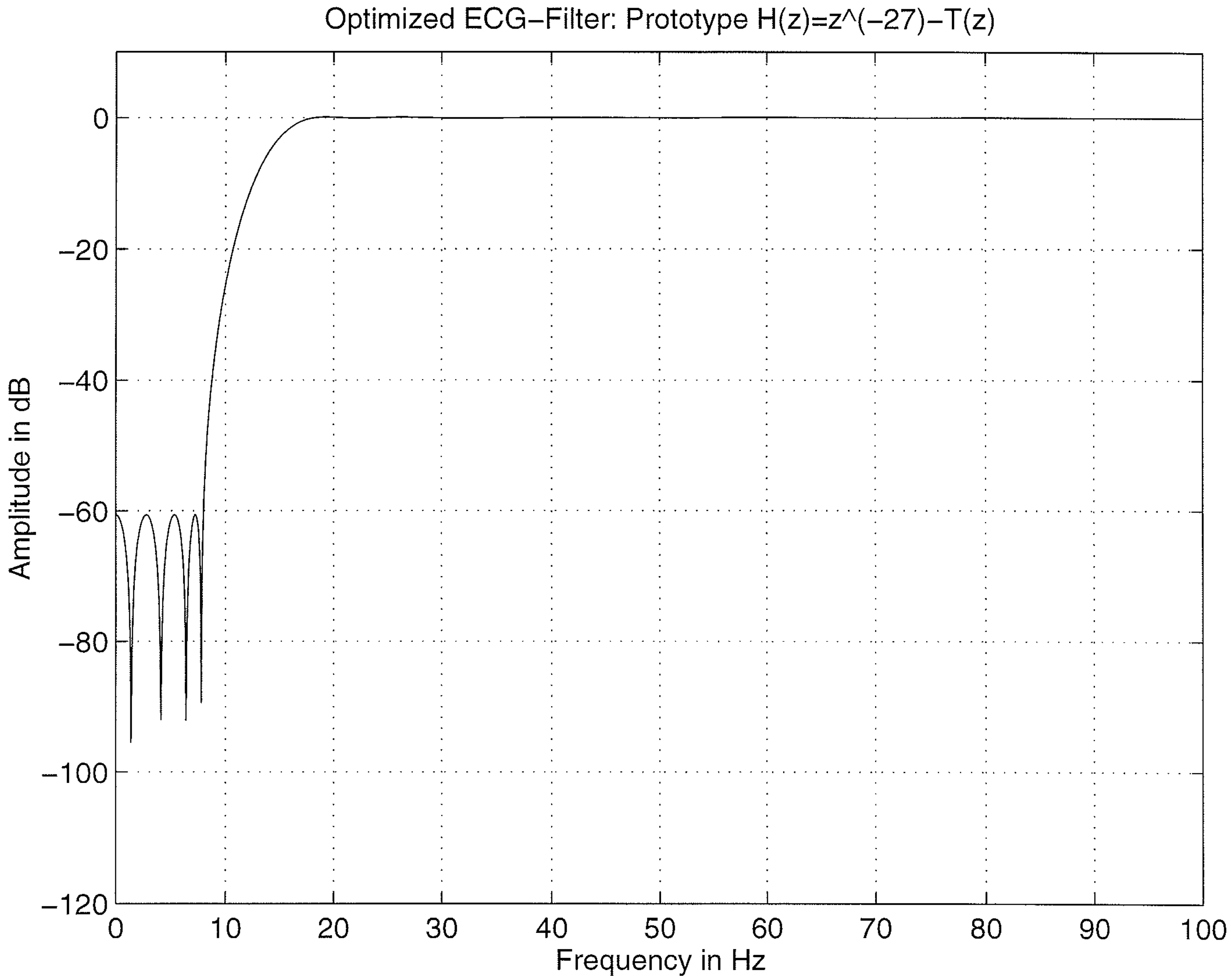

New ECG−Filter: H(z^4)
Impulse response
n in samples
1
0.8
0.6
0.4
0.2
0
−0.2
0
50
100
150
200
250

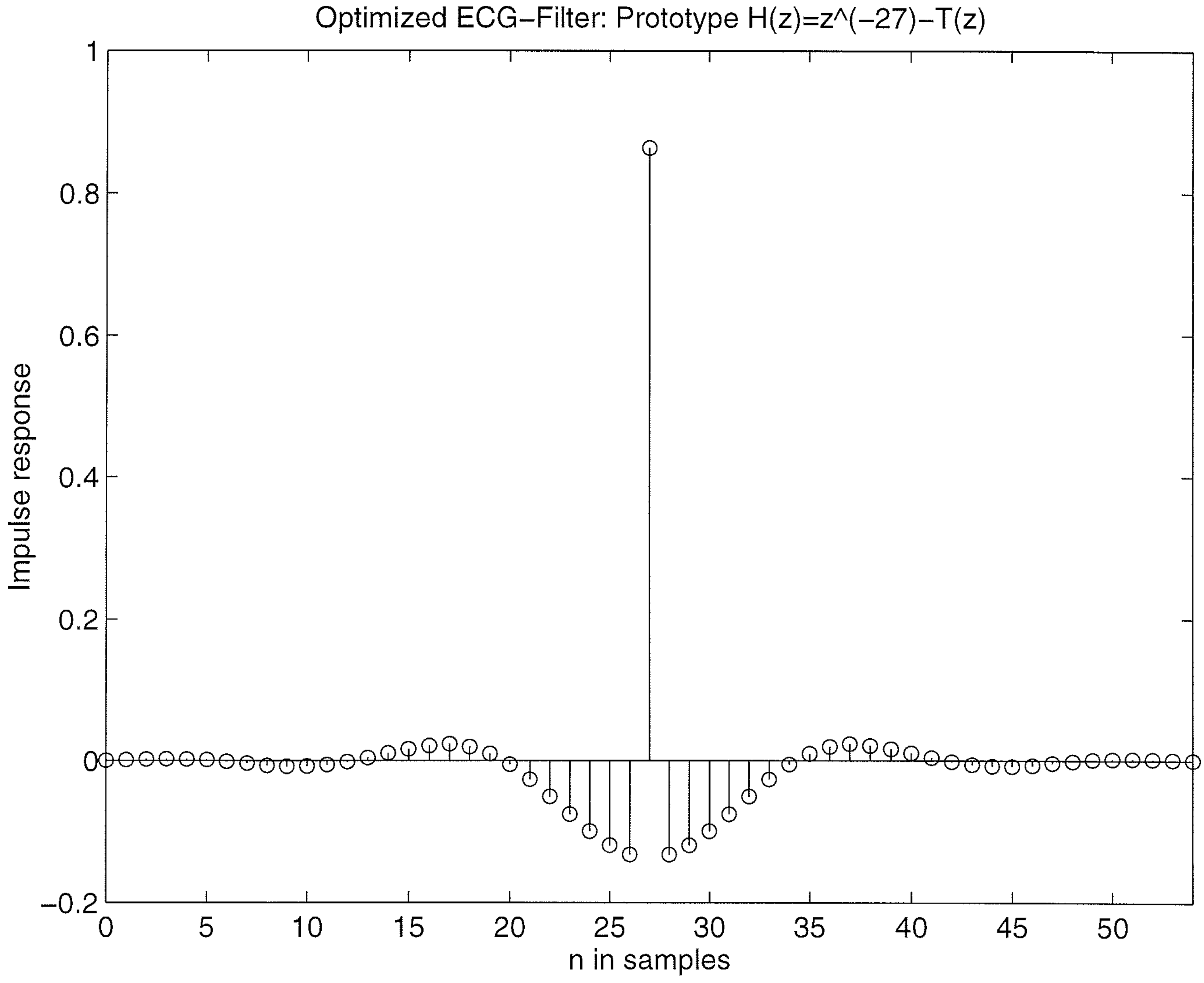
ECG FILTER: OPTIMIZED DESIGN

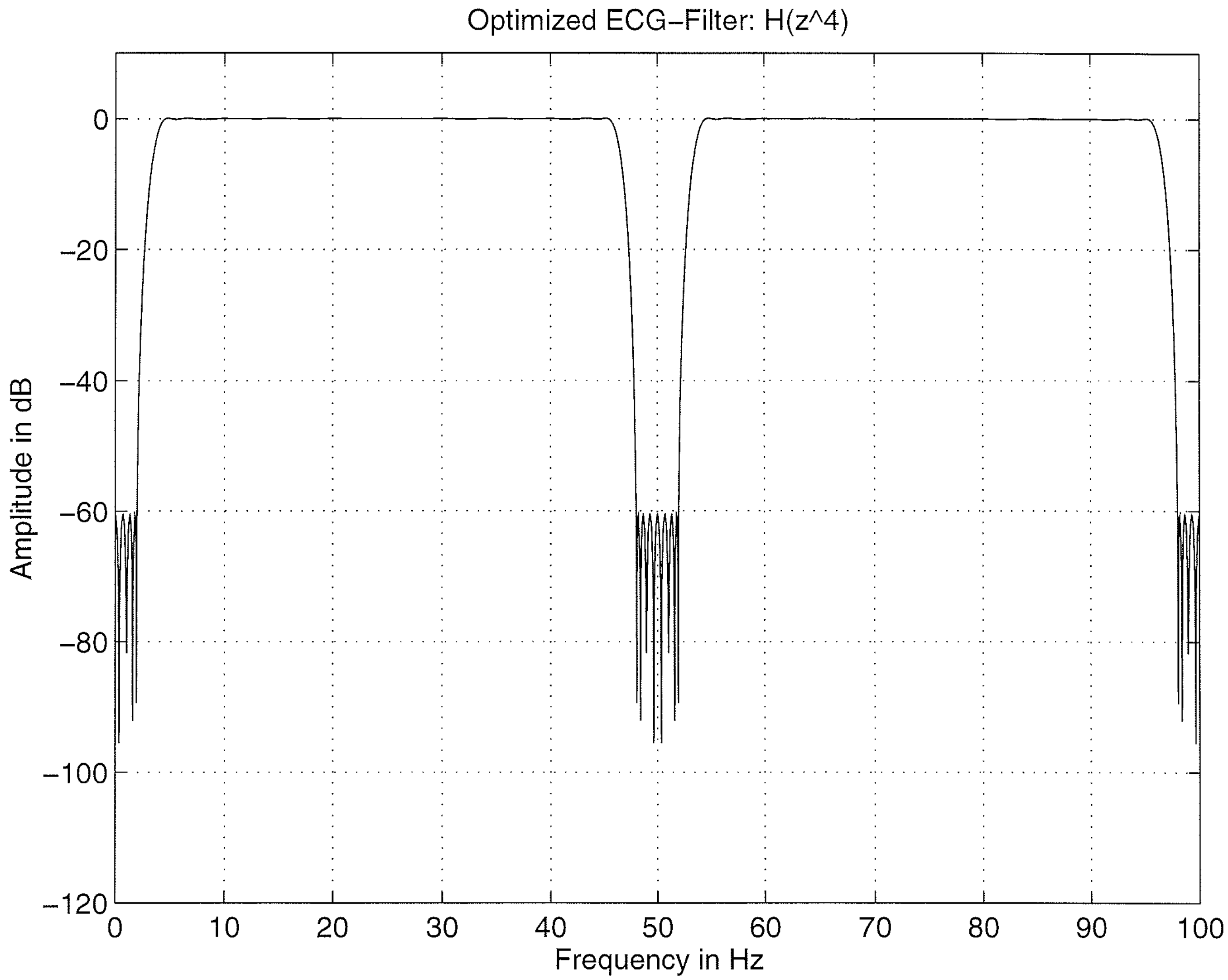
- Here, we design $F(z^3)$ and $G(z)$ using the algorithm described in T. Saramäki, "Finite impulse response Filter Design" in Handbook for Digital Signal Processing, S. K. Mitra and J. F. Kaiser, Eds., John Wiley & Sons, 1993, pp. 241-245.
- Using this algorithm, the orders of $F(z)$ and $G(z)$ reduce from 19 to 16 and from 11 to 6.
- In this case, $H(z^4) = z^{-108} - F(z^{12})G(z^4)$. The overall order is 216, which is only slightly larger than that (208) of the old design. The number of multipliers reduces to $16 + 1 + 6 + 1 = 24$, which is less than half that (53) required by the old design.
- In the following there are six transparencies illustrating the performance of this design.

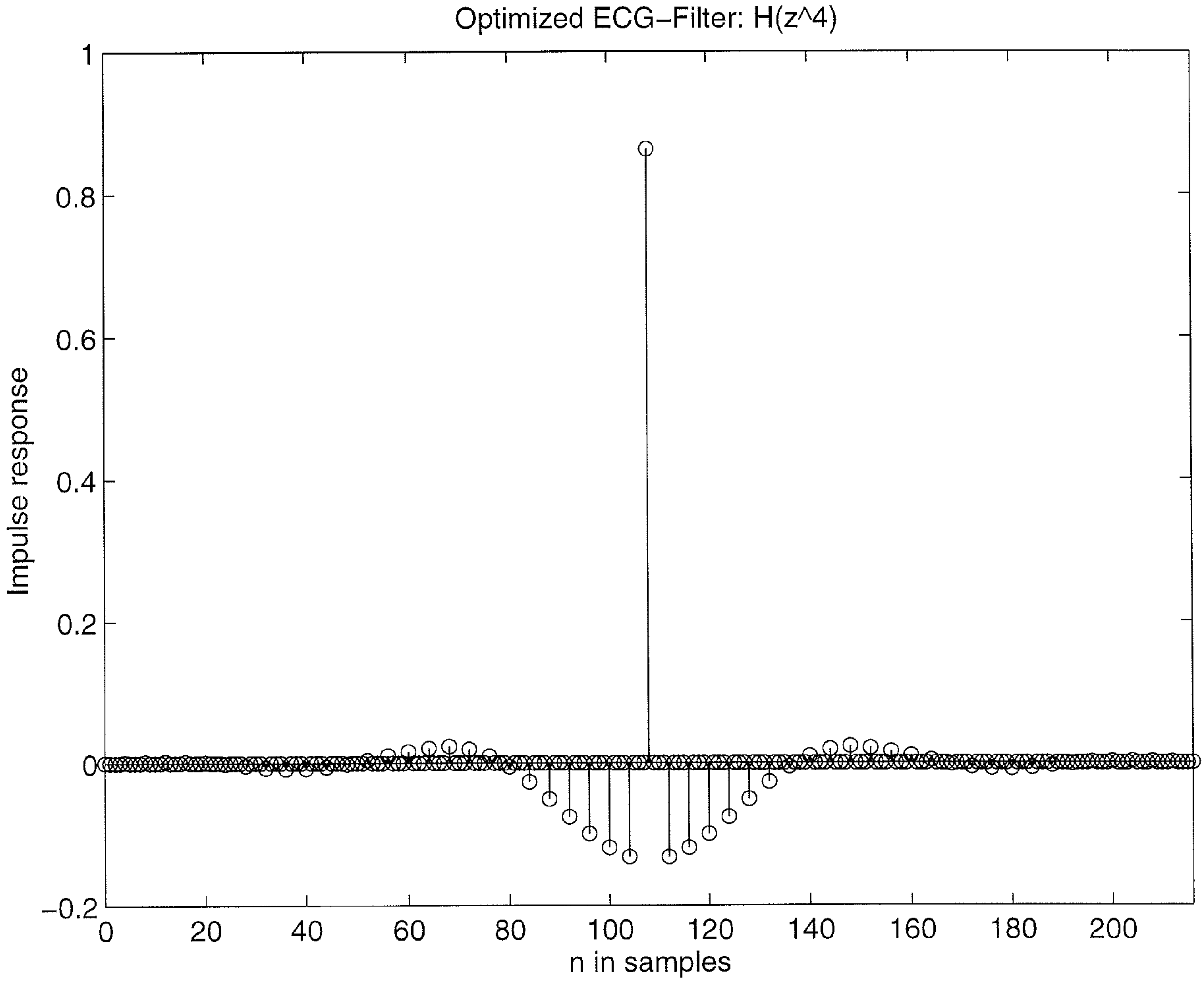

Components for T(z)=F(z^3)G(z): F(z) and G(z) of orders 16 and 6
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz


Optimized ECG−Filter: T(z): complementary for the prototype filter
Amplitude in dB
Frequency in Hz
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100


Optimized ECG−Filter: Prototype H(z)=z^(−27)−T(z)
0
−20
−40
−60
−80
−100
−120
Amplitude in dB
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz


Optimized ECG−Filter: Prototype H(z)=z^(−27)−T(z)
Impulse response
1
0.8
0.6
0.4
0.2
0
−0.2
0
5
10
15
20
25
30
35
40
45
50
n in samples


Optimized ECG−Filter: H(z^4)
0
−20
−40
−60
−80
−100
−120
Amplitude in dB
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

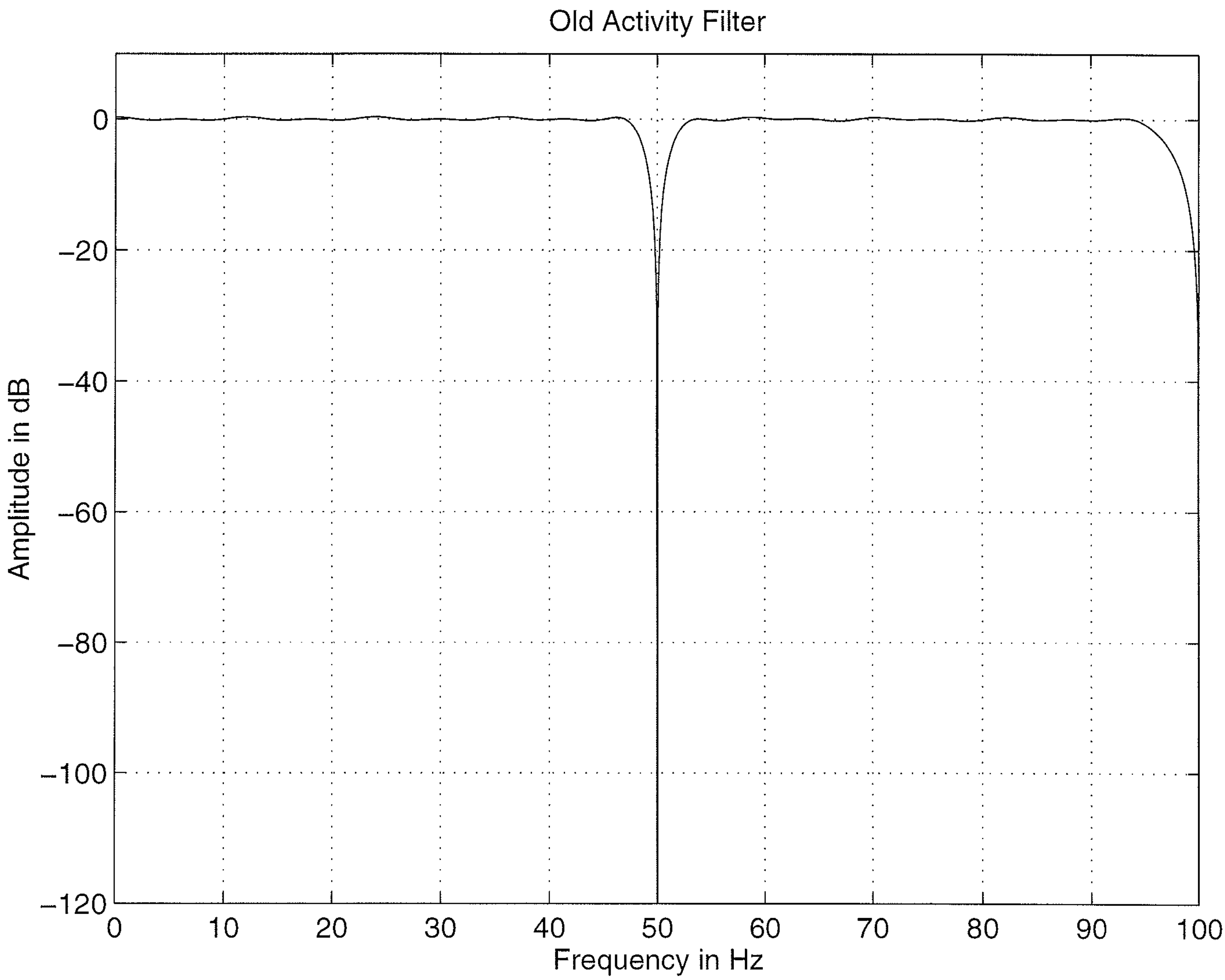

Optimized ECG–Filter: H(z^4)
Impulse response
n in samples
1
0.8
0.6
0.4
0.2
0
−0.2
0
20
40
60
80
100
120
140
160
180
200

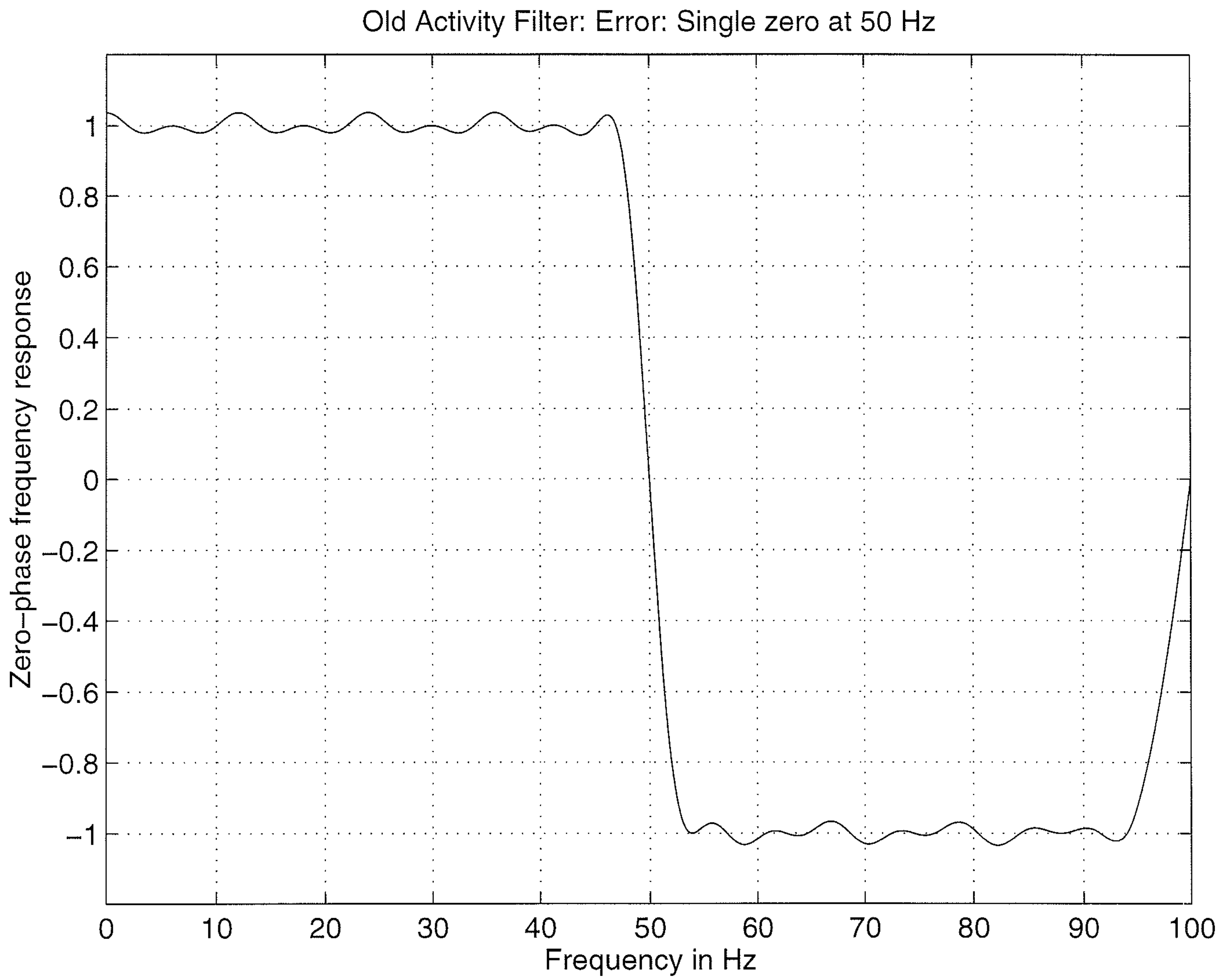
ACTIVITY FILTER: OLD DESIGN

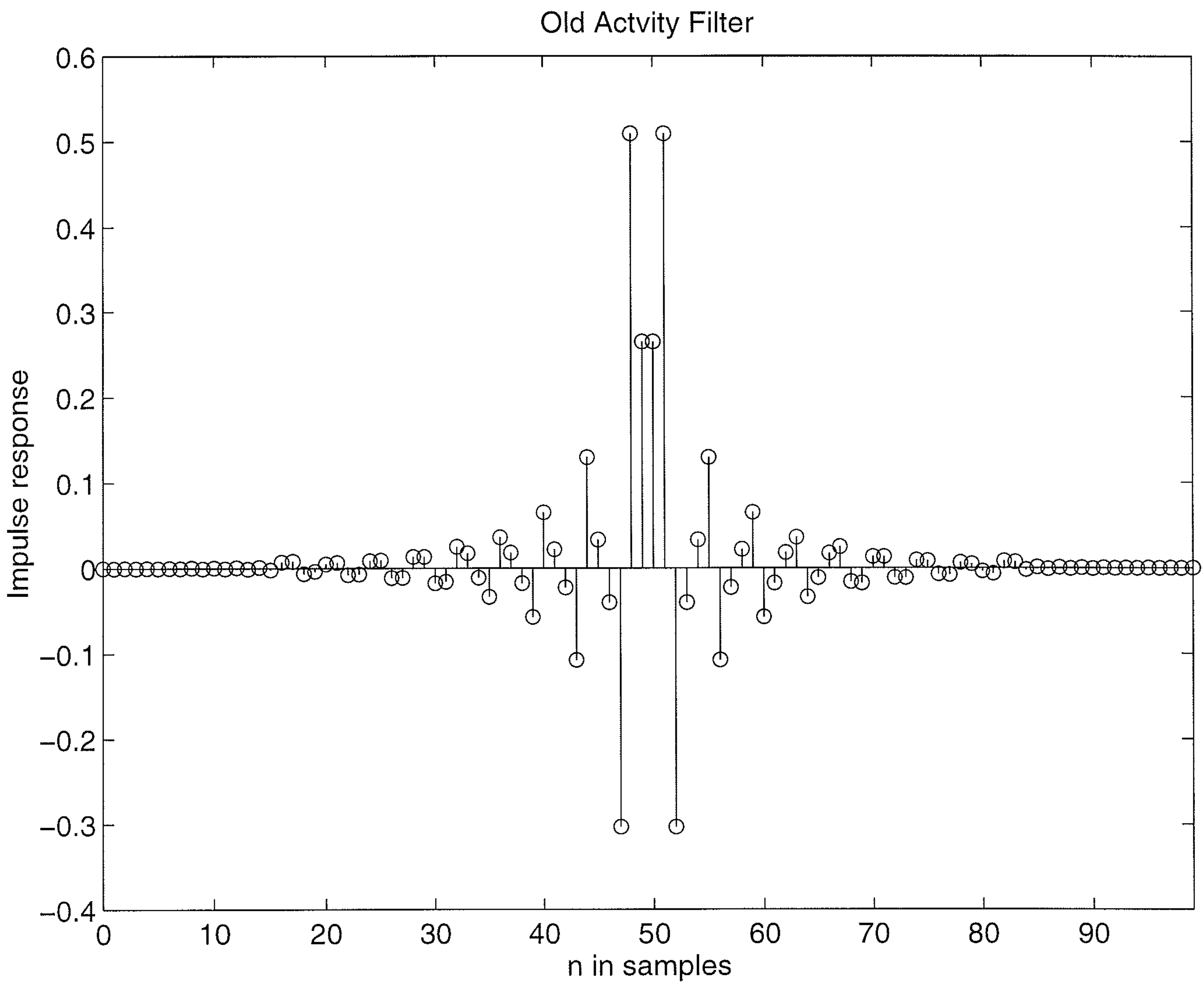
- It is desired to design for $f_s = 200$ Hz a filter $H(z)$ having zeros at 50 Hz and at 100 Hz to remove the line frequency interference. The passband regions are from 0 Hz to 47 Hz and from 53 Hz to 94 Hz. The passband ripple is $\delta_p = 0.01$.
- In the article in the end of this pile, they first design for $f_s = 200$ a filter $G(z)$ of order 33 such that the passband region is from 0 Hz to 94 Hz. For the Remez-algorithm, there is only one passband $[0, 0.94\pi]$ and the desired and weighting functions are unity (see Matlab-file fishi2.m). Because of an odd order, there is automatically a zero at $z = -1$ (at $\omega = \pi$ or at 100 Hz).
- The periodic filter $G(z^2)$ has then for $f_s = 200$ a zero at 50 Hz and passband regions from 0 Hz to 47 Hz and from 53 Hz to 100 Hz.
- Filter $H(z) = G(z^2)G(z)$ has the zeros at the right positions and the desired passband regions.
- In the following there are three transparencies illus-

trating the performance of this design.

- The overall design suffers, however, from a serious problem. As seen from the figure giving the zero-phase frequency response of the overall design, the zero-phase response changes its sign at 50 Hz.
- This means that the phase response has a jump of π at 50 Hz. Because of this jump, the phase delay is not constant in the interval from 0 Hz to 100 Hz. Therefore, the filter is useless in the cases where the waveforms are desired to be preserved.


Old Activity Filter
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz


Old Activity Filter: Error: Single zero at 50 Hz
Zero−phase frequency response
Frequency in Hz
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
10
20
30
40
50
60
70
80
90
100


Old Actvity Filter
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
Impulse response
0
10
20
30
40
50
60
70
80
90
n in samples

ACTIVITY FILTER: NEW DESIGN

- The filter is designed in the following steps:

1) For $f_s = 200$ Hz, we design a filter $T(z)$ such that it achieves the value of unity at 0 Hz and its attenuation is at least 40 dB ($\delta_s = 0.01$) in the range from $4 \cdot 3 = 12$ Hz to 100 Hz. In the ω-scale, $\omega_s = 0.12\pi$. Our filter transfer function is of the form

$$T(z) = F(z^4)[RRS4(z)]^2,$$

where

$$RRS4(z) = 1/4 + 1/4z^{-1} + 1/4z^{-2} + 1/4z^{-3}.$$

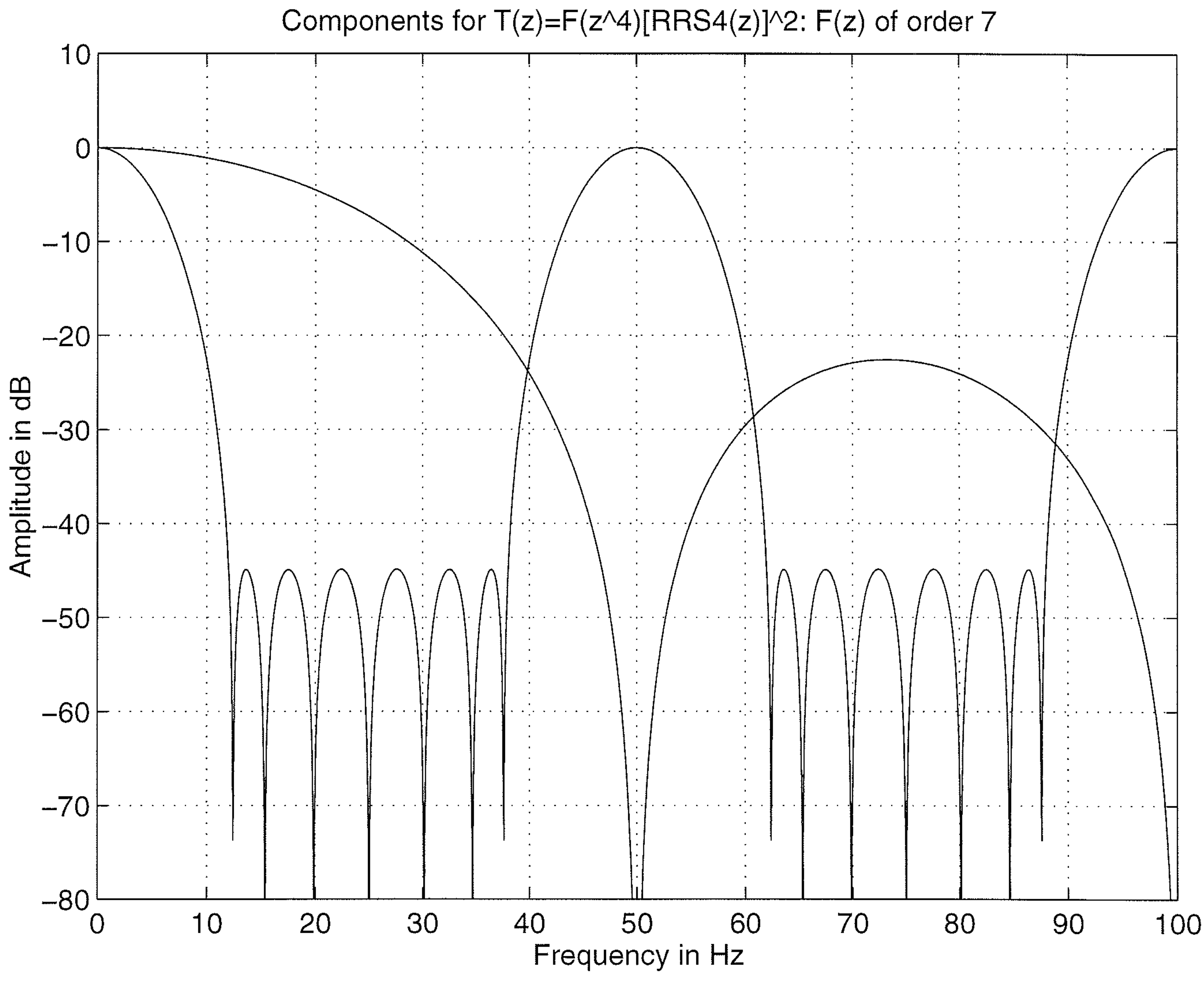
For $F(z)$, the stopband edge is $4\omega_s = 0.48\pi$. The given criteria are met by $F(z)$ of order 7.

2) $T(z^4)$ is periodic achieving the value of unity for $f_s = 100$ Hz at 0, 50, and 100 Hz. The stopband regions are from 0 Hz to 47 Hz and from 53 Hz to 97 Hz.

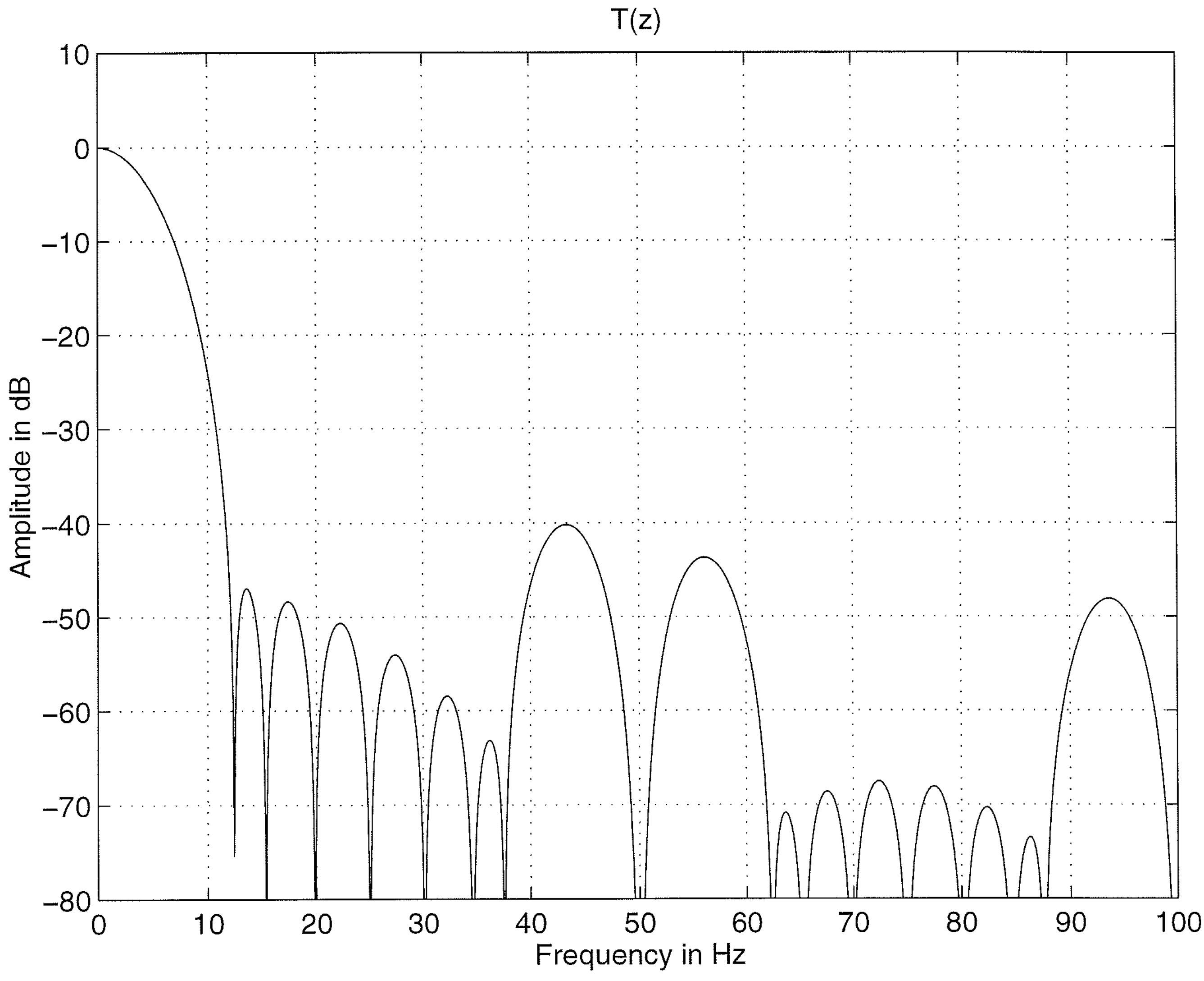
3) Cascade $T(z^4)$ with $z^{-3} - [RRS4(z)]^2$, giving $S(z) = T(z^4)[z^{-3} - [RRS4(z)]^2]$. Here, $z^{-3} - [RRS4(z)]^2$ attenuates the first passband region of $T(z^4)$ while still keeping the value of unity at 50 and 100 Hz ($z^{-3} - [RRS4(z)]^2$ achieves the value of unity at these points.)

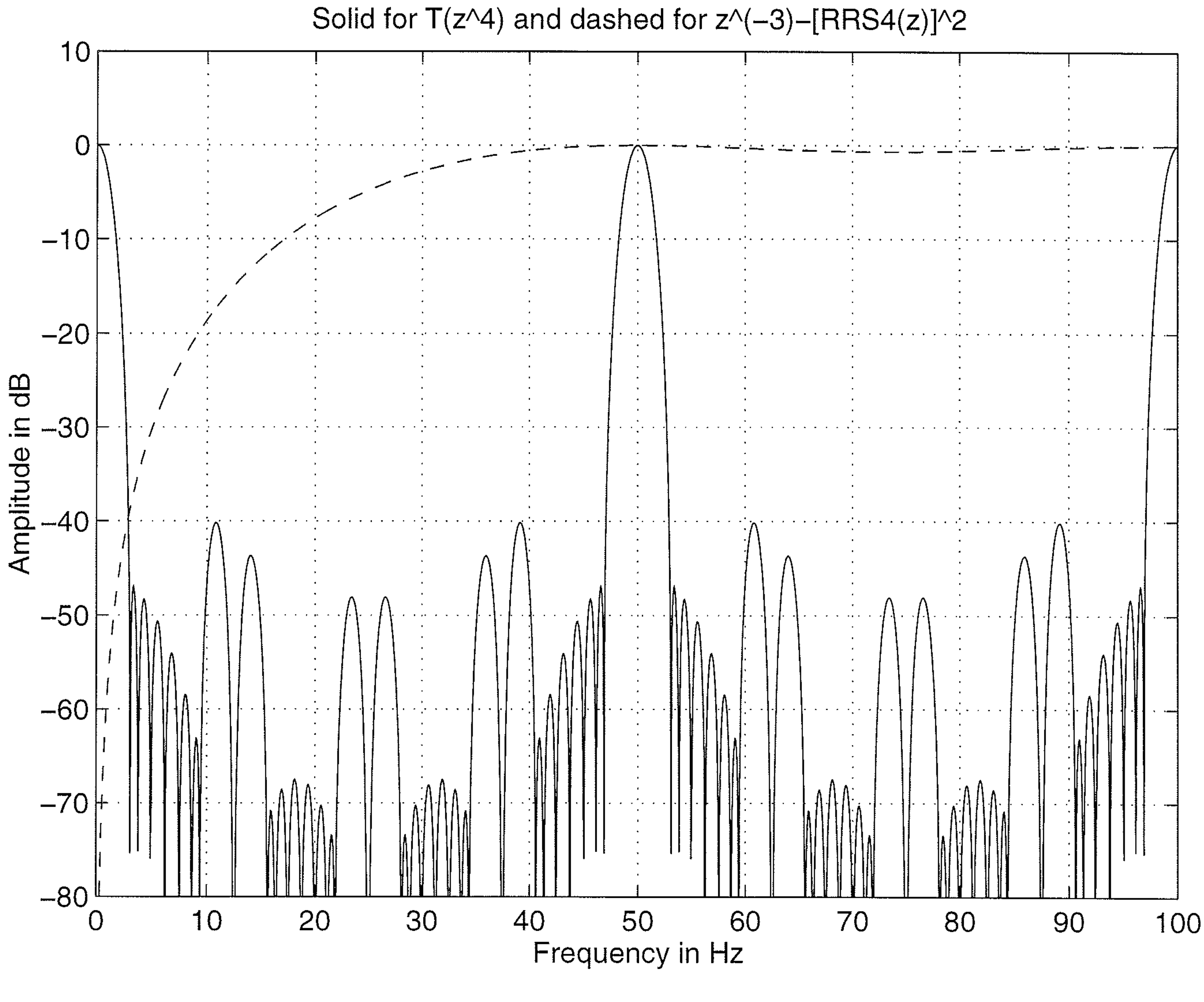
4) The desired overall filter is then $z^{-71} - S(z)$ providing zeros at 50 Hz and 100 Hz. The passband regions are from 3 Hz to 47 Hz and from 53 Hz to 97 Hz.

- In the following there are altogether 7 transparencies illustrating the performance of this design. One of them shows that the zero-phase frequency response of the overall filter. This response does not change its sign at 50 Hz so that the phase is linear, as is desired.

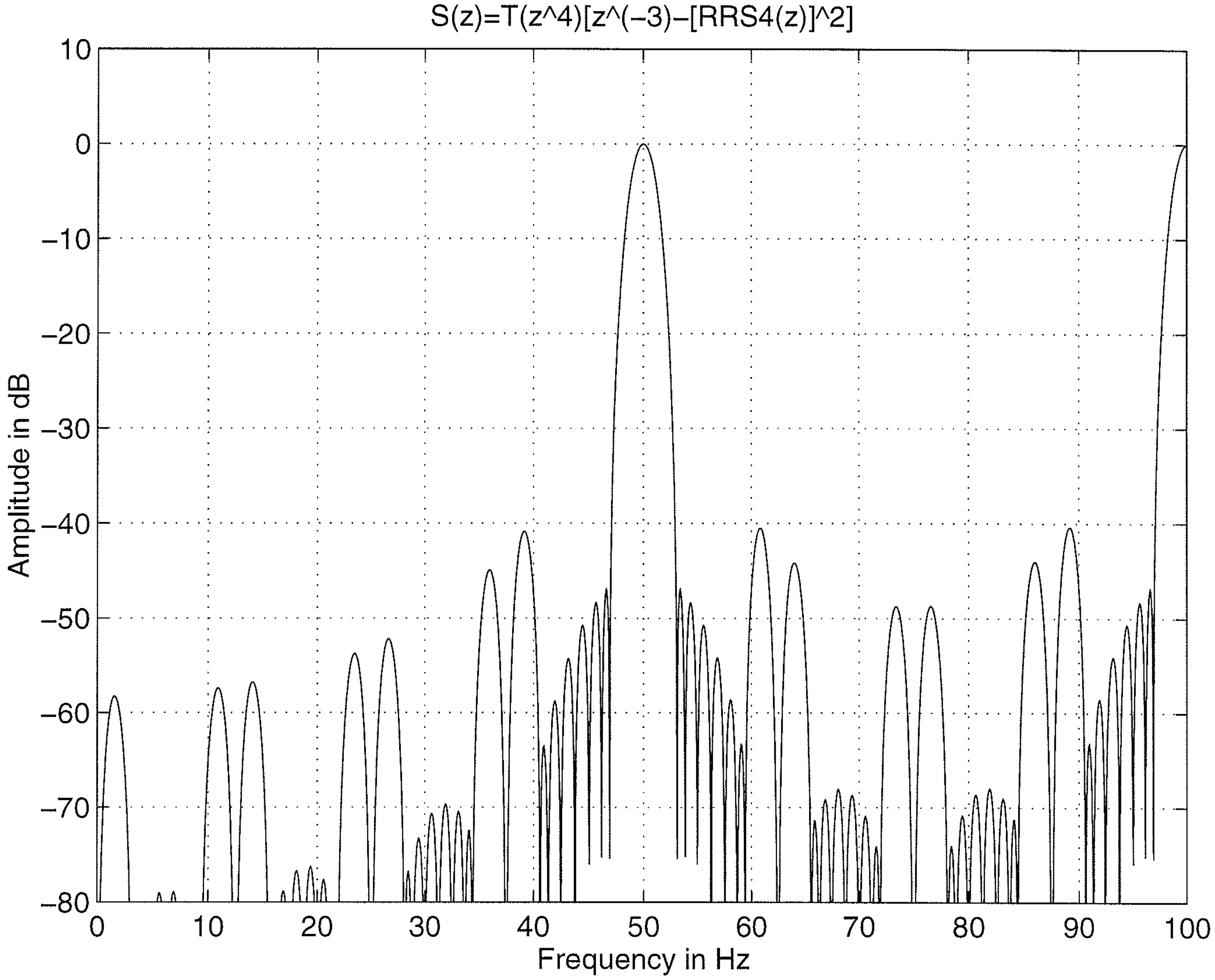


Components for T(z)=F(z^4)[RRS4(z)]^2: F(z) of order 7
Amplitude in dB
Frequency in Hz
10
0
−10
−20
−30
−40
−50
−60
−70
−80
0
10
20
30
40
50
60
70
80
90
100

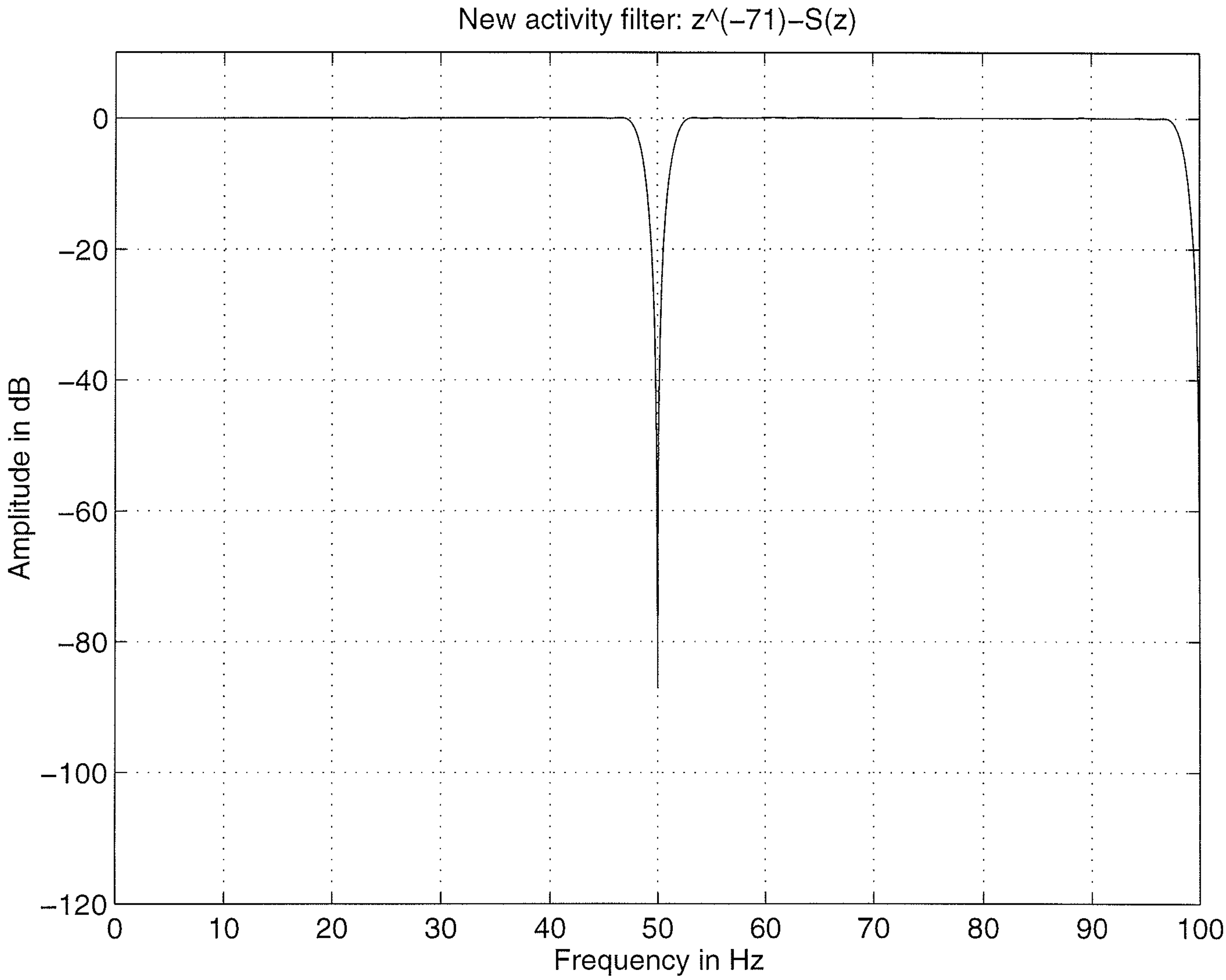

T(z)
Amplitude in dB
Frequency in Hz
10
0
−10
−20
−30
−40
−50
−60
−70
−80
0
10
20
30
40
50
60
70
80
90
100



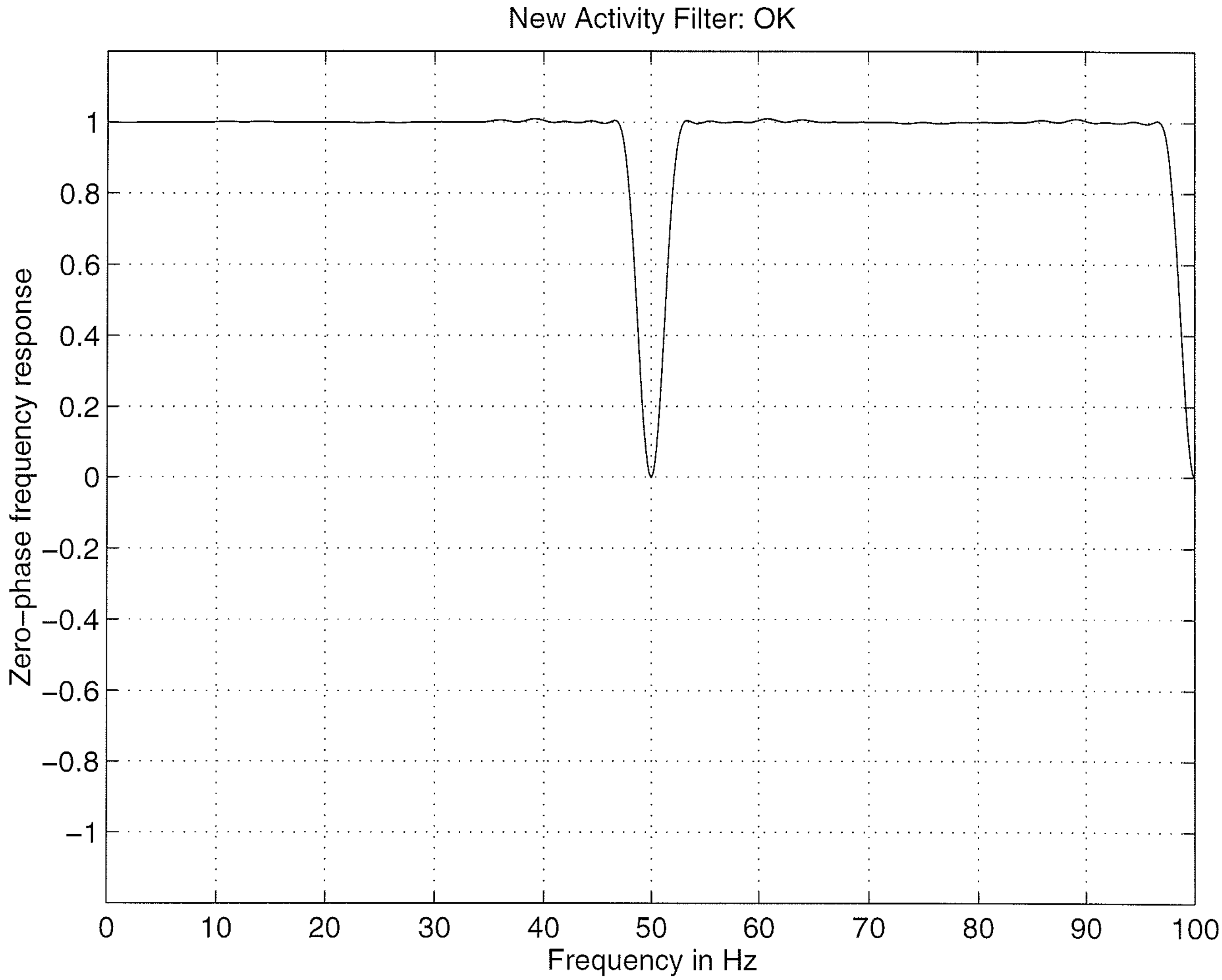
Solid for T(z^4) and dashed for z^(−3)−[RRS4(z)]^2
Amplitude in dB
Frequency in Hz
10
0
−10
−20
−30
−40
−50
−60
−70
−80
0
10
20
30
40
50
60
70
80
90
100

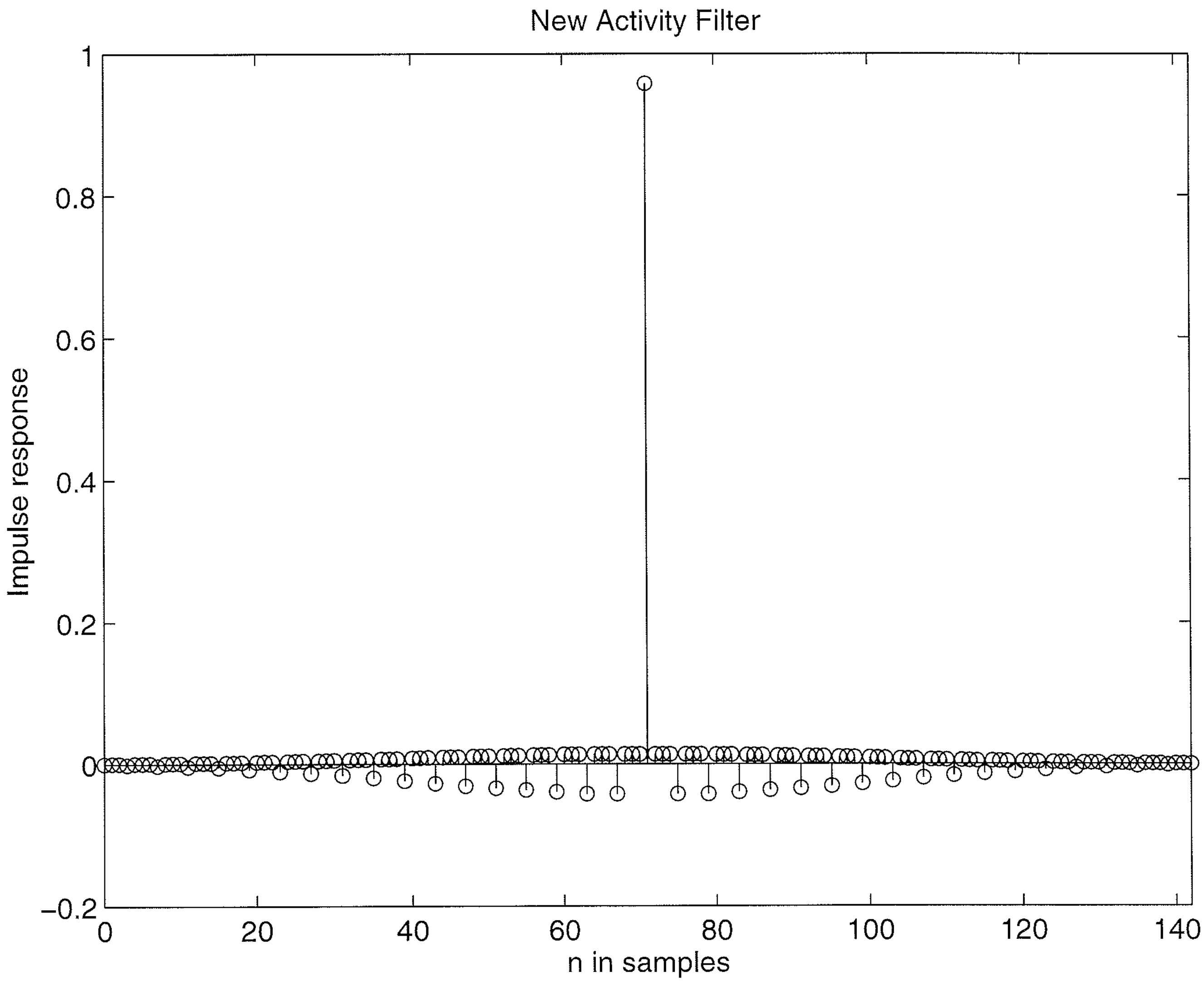


S(z)=T(z^4)[z^(−3)−[RRS4(z)]^2]
Amplitude in dB
Frequency in Hz
10
0
−10
−20
−30
−40
−50
−60
−70
−80
0
10
20
30
40
50
60
70
80
90
100



New activity filter: z^(−71)−S(z)
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

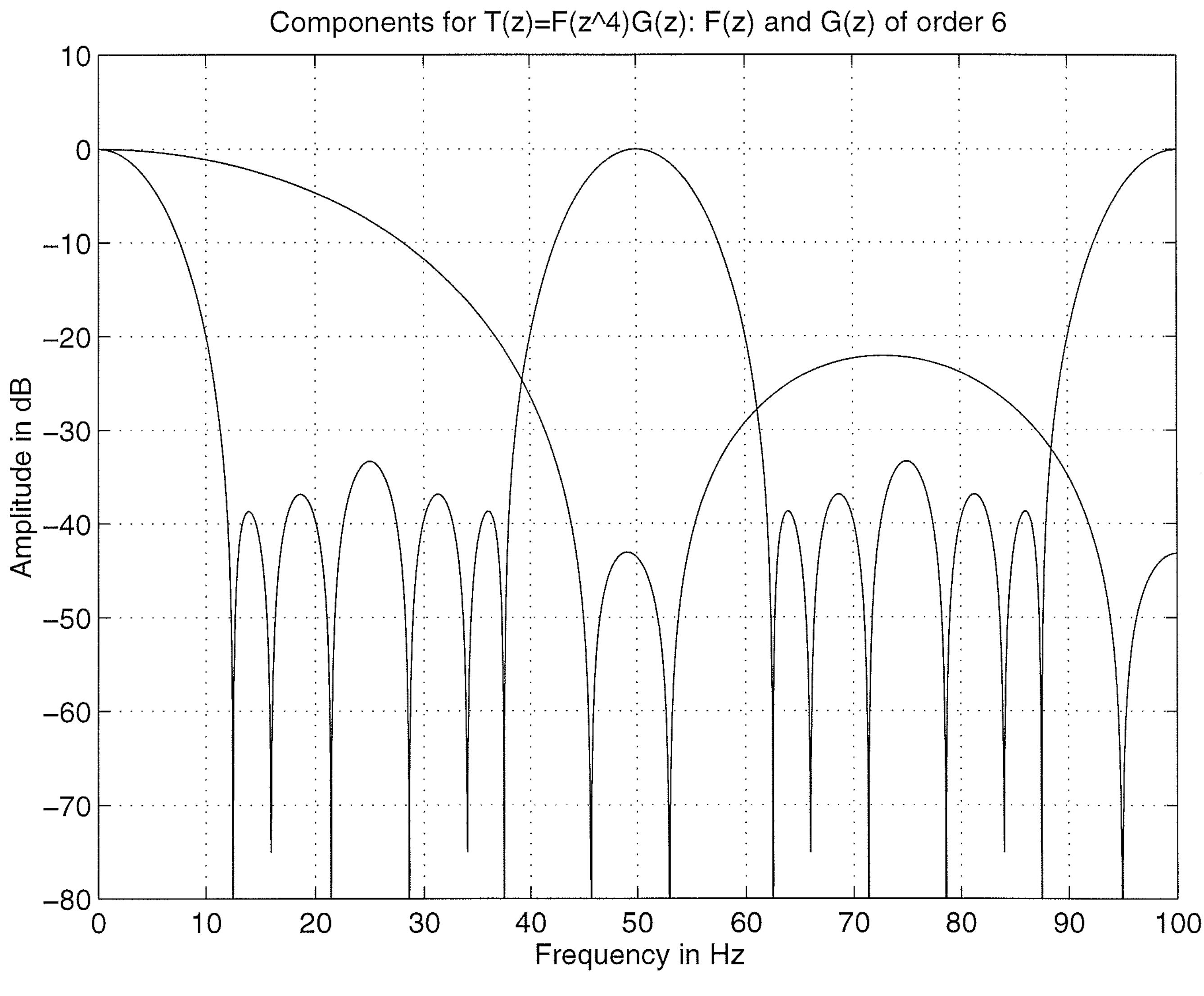

New Activity Filter: OK
Zero−phase frequency response
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

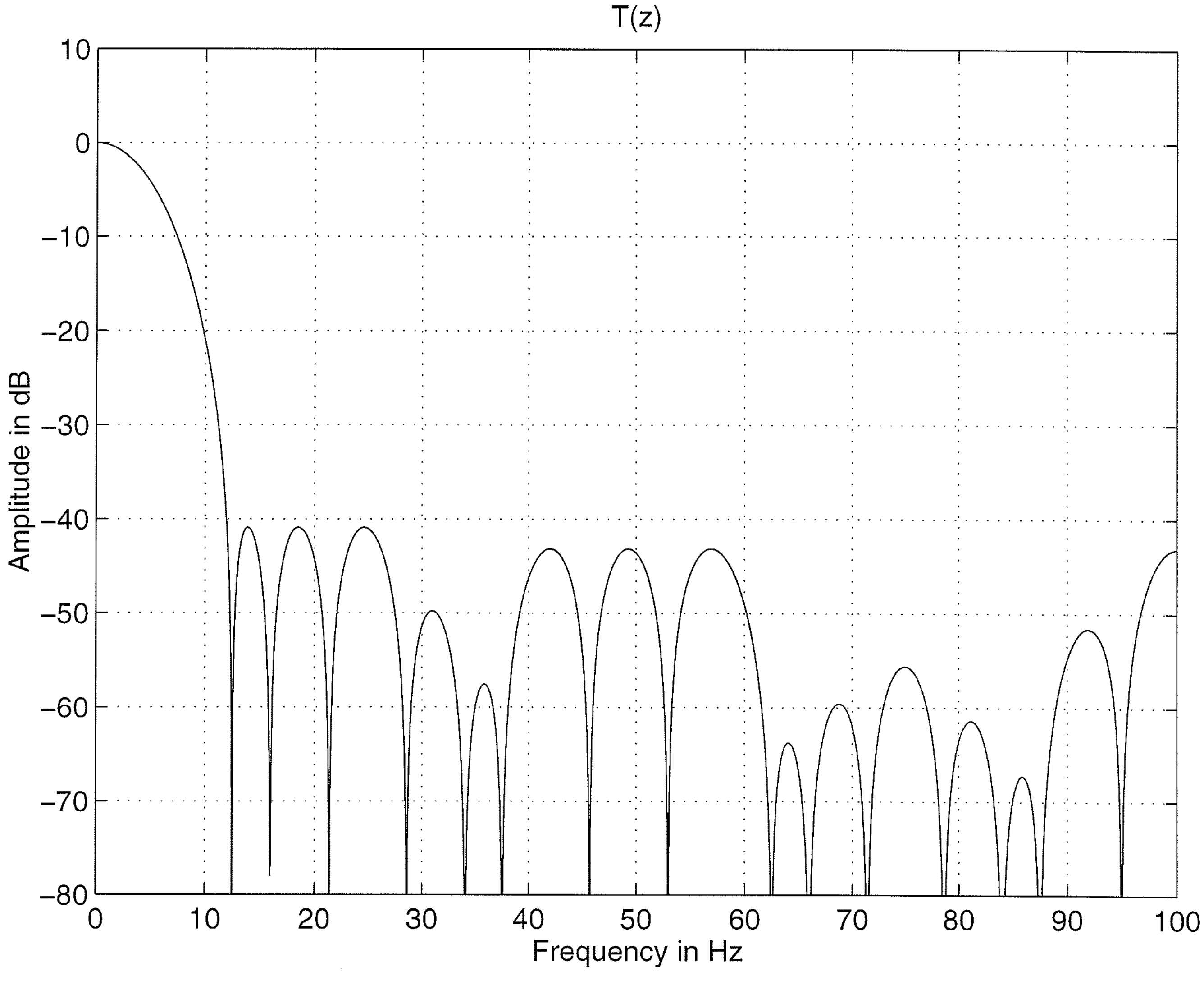


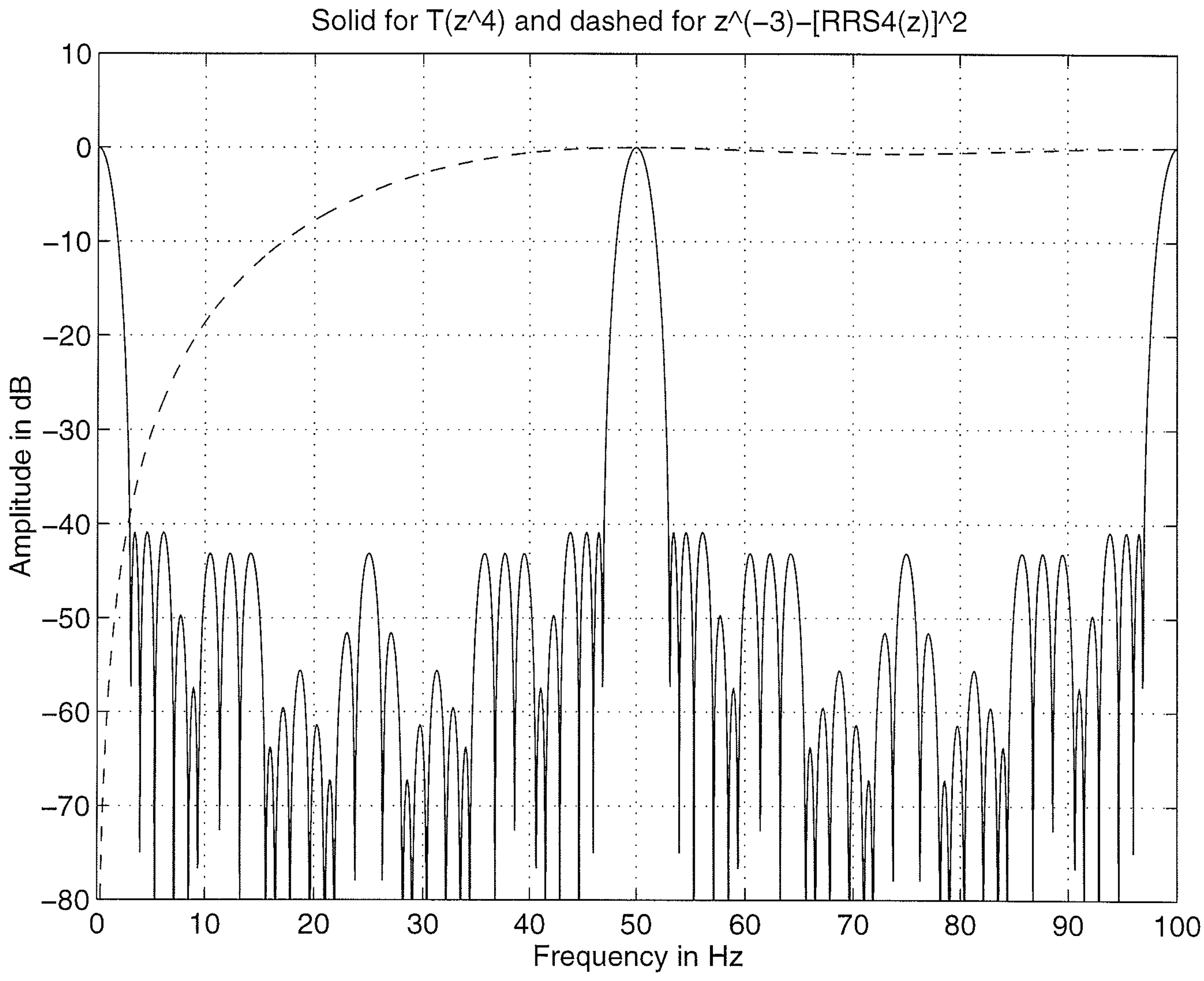
New Activity Filter
Impulse response
n in samples
1
0.8
0.6
0.4
0.2
0
−0.2
0
20
40
60
80
100
120
140

ACTIVITY FILTER: OPTIMIZED DESIGN

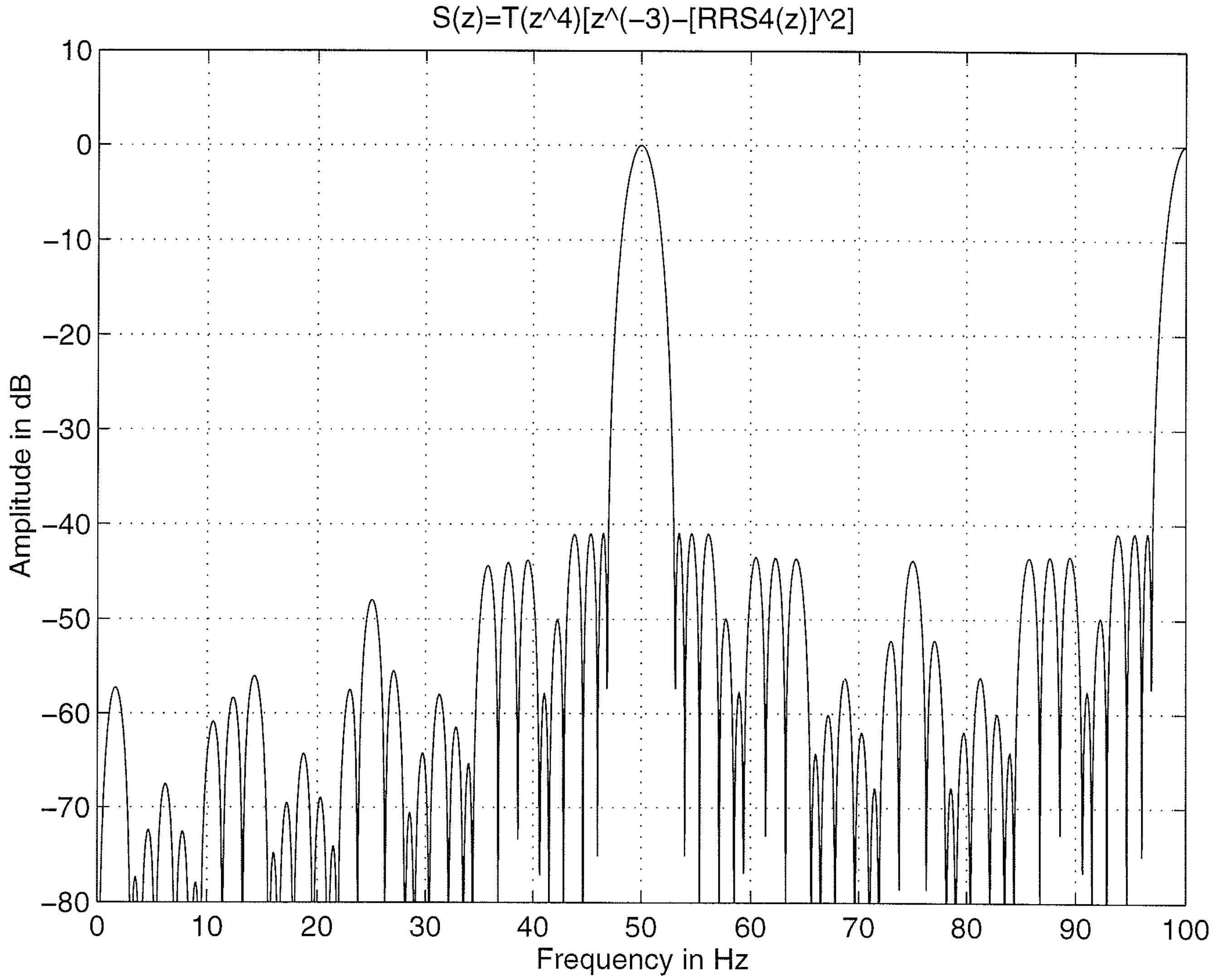
- Here, we design $T(z)$ in the form $T(z) = F(z^4)G(z)$ and $G(z)$ using the algorithm described in T. Saramäki, "Finite impulse response Filter S. K. Mitra and J. F. Kaiser, Eds, John Wiley & Sons, 1993, pp. 241-245. Otherwise the design is the same.
- Using this algorithm, the orders of both $F(z)$ and $G(z)$ are 6.
- In the following there are altogether 7 transparencies illustrating the performance of this design. One of them shows that the zero-phase frequency response of the overall filter. This response does not change its sign at 50 Hz so that the phase is linear, as is desired.

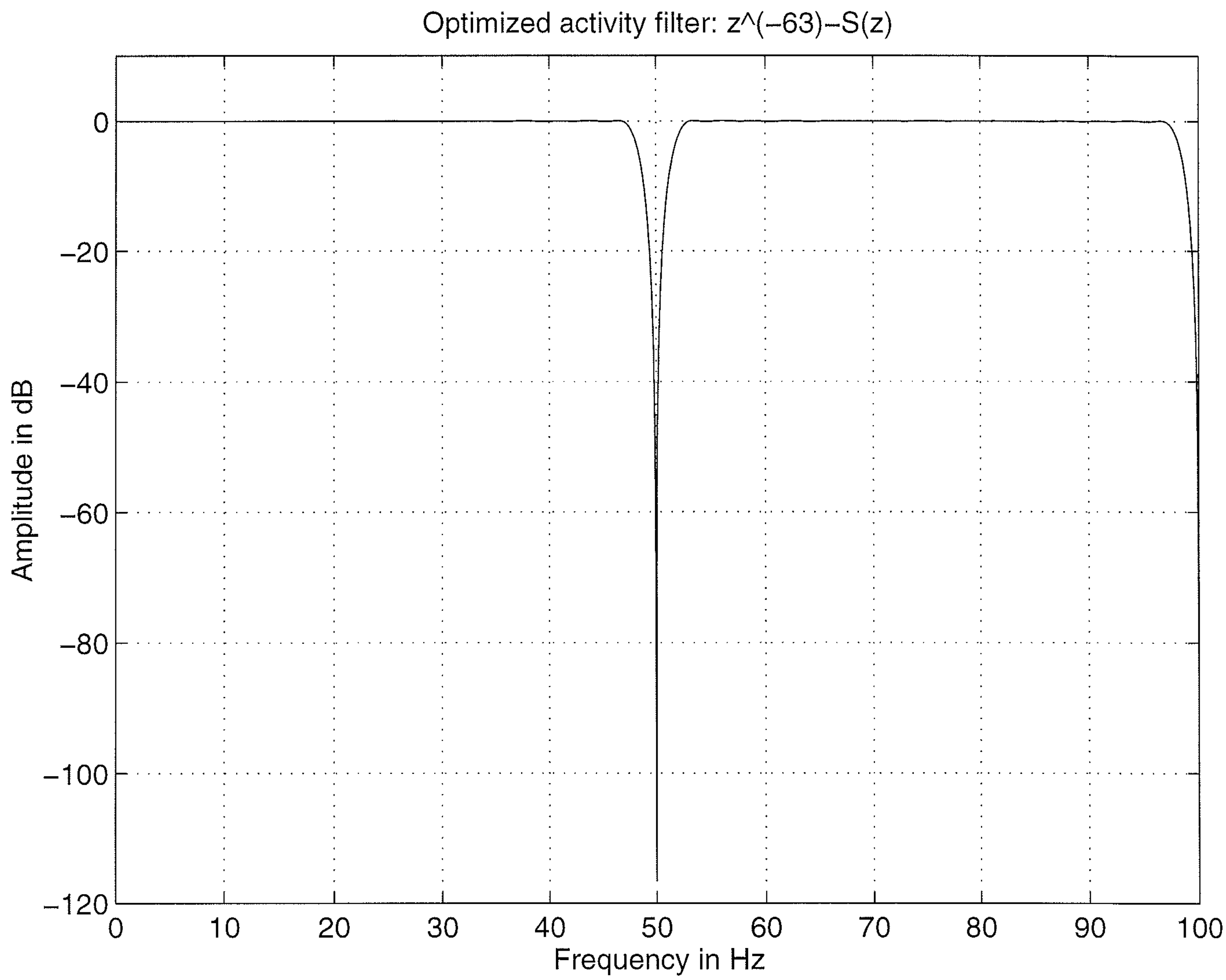

Components for T(z)=F(z^4)G(z): F(z) and G(z) of order 6
Amplitude in dB
10
0
−10
−20
−30
−40
−50
−60
−70
−80
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz


T(z)
Amplitude in dB
Frequency in Hz
10
0
−10
−20
−30
−40
−50
−60
−70
−80
0
10
20
30
40
50
60
70
80
90
100

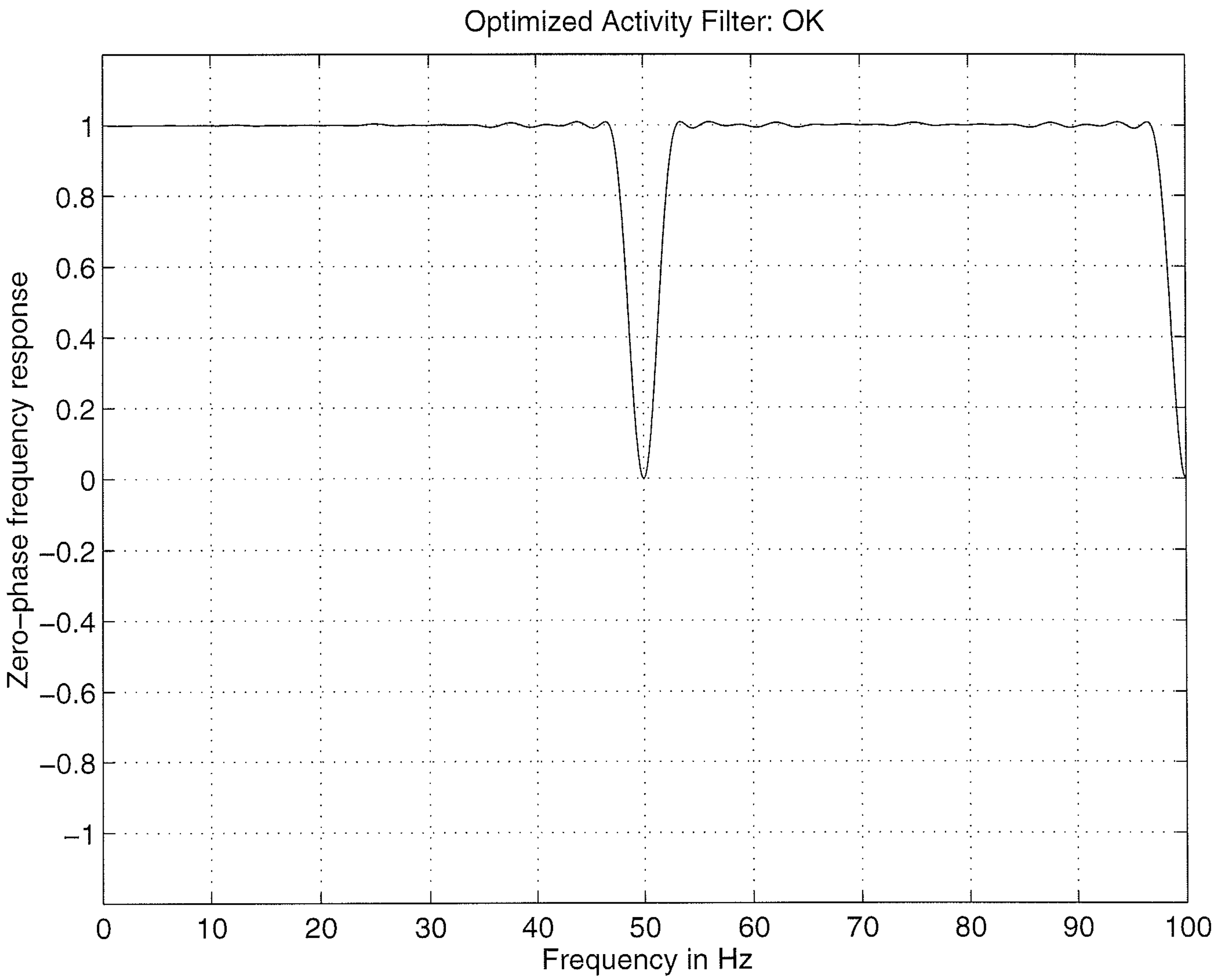


Solid for T(z^4) and dashed for z^(−3)−[RRS4(z)]^2
10
0
−10
−20
−30
−40
−50
−60
−70
−80
Amplitude in dB
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz

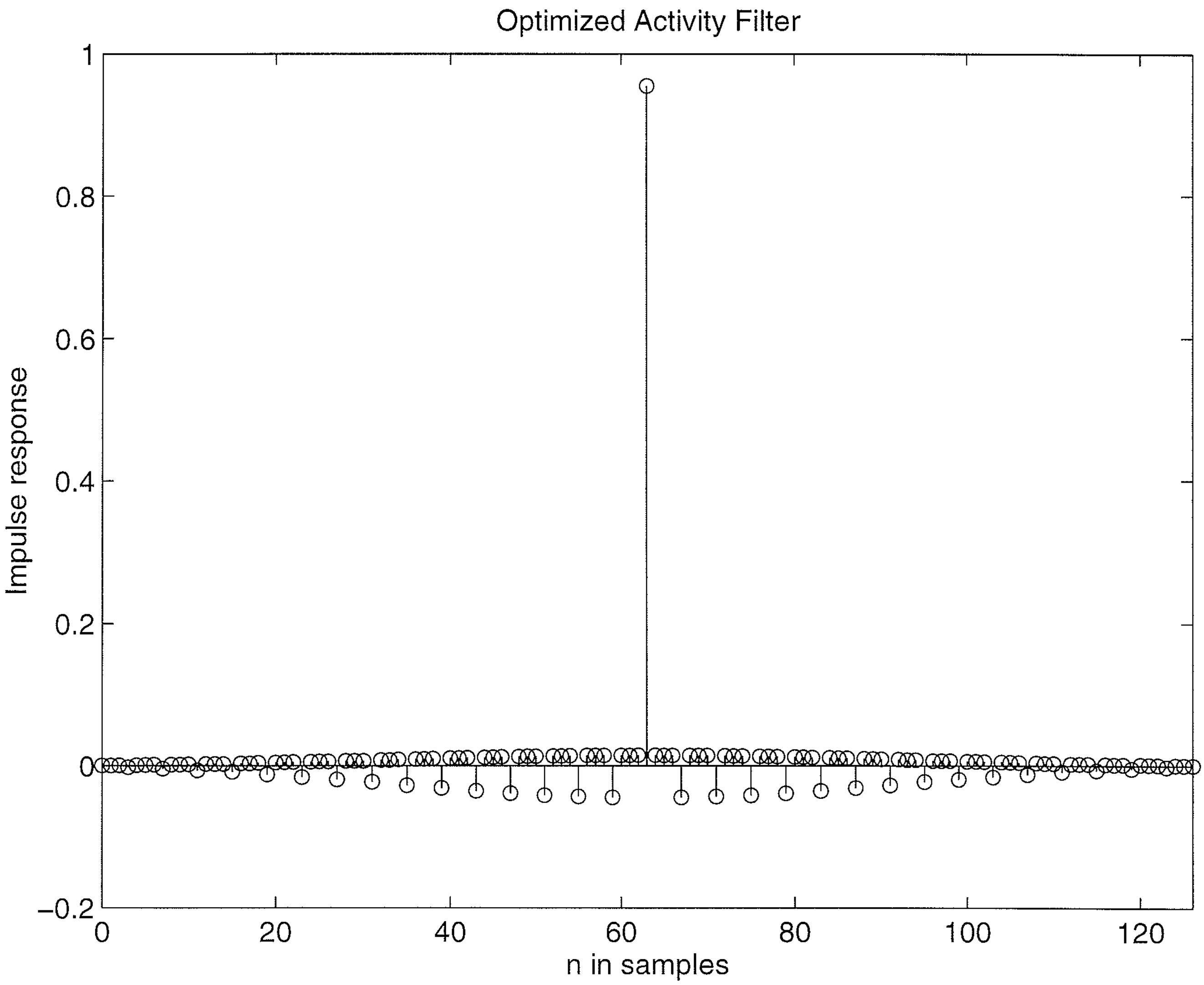

S(z)=T(z^4)[z^(-3)-[RRS4(z)]^2]
Amplitude in dB
Frequency in Hz
10
0
-10
-20
-30
-40
-50
-60
-70
-80
0
10
20
30
40
50
60
70
80
90
100



Optimized activity filter: z^(−63)−S(z)
Amplitude in dB
0
−20
−40
−60
−80
−100
−120
0
10
20
30
40
50
60
70
80
90
100
Frequency in Hz



Optimized Activity Filter: OK
Zero-phase frequency response
Frequency in Hz
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
10
20
30
40
50
60
70
80
90
100


Optimized Activity Filter
Impulse response
n in samples
1
0.8
0.6
0.4
0.2
0
−0.2
0
20
40
60
80
100
120

```
% Matlab-file fishi1.m
%
%%%%%%%%%%%%%%%%%%%%%%%%%%
%ECG-Filter: old version
%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%%%%%%%
%First ptototype H(z)
%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
f=[0. 8 18 100];
f=f/100;
m=[0 0 1 1];
w=[10 1];
h=remez(52, f, m, w);
figure(1)
[H,W]=zeroam(h,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('Old ECG-Filter: Prototype H(z) of order 52')
figure(2)
impz(h)
xlabel('n in samples');
ylabel('Impulse response');
title('Old ECG-Filter: Prototype H(z) of order 52')
%%%%%%%%%%%%%%%%%%%%%%%%%
%Then H(z^4)
%%%%%%%%%%%%%%%%%%%%%%%%%
for k=1:53 hh(4*k-3)=h(k);end
figure(3)
[H,W]=zeroam(hh,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('Old ECG-Filter: H(z^4)')
figure(4)
impz(hh)
xlabel('n in samples');
ylabel('Impulse response');
title('Old ECG-Filter: H(z^4)')
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%New ECG Prototype Filter: H(z)=z^(-M)-T(z)
%T(z)=F(z^3)G(z), M=half the order of T(z)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%For highpass H(z) the stopband edge is 8*pi/100=0.08pi
%and the passband edge is 18*pi/100=0.18pi
%stopband and passband ripples are 0.001 and 0.01
%T(z)=F(z^3)G(z) is a complementary lowpass filter
%It can be designed such that F(z) is a lowpass
%filter with edges at 0.24*pi and 0.54*pi
%and ripples of 0.0005 and 0.01
%G(z) has the same ripple values, passband region is
%[0, 0.08*pi] and stopband region is
%[2/3-(2*.18+.08)/3 2/3+(2*.18+.08)/3].
%
%Design F(z)
```

```
%
f=[0. .24 .54 1];
m=[1 1 0 0];
w=[20 1];
ff=remez(19, f, m, w);
%
%Design G(z)
%
f=[0. .08 2/3-(2*.18+.08)/3 2/3+(2*.18+.08)/3];
m=[1 1 0 0];
w=[20 1];
gg=remez(11, f, m, w);
%
%Impulse response of T(z)=F(z^3)G(z)
%
for k=1:20 fff(3*k-2)=ff(k);end
t=conv(fff,gg);
[H,W]=zeroam(fff,.0,1.,2000);
[HH,W]=zeroam(gg,.0,1.,2000);
figure(5)
plot(100*W/pi,20*log10(abs(H)),100*W/pi,20*log10(abs(HH)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('Components for T(z)=F(z^3)G(z): F(z) and G(z) of orders 19 and 11')
figure(6)
[H,W]=zeroam(t,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('New ECG-Filter: T(z): complementary for the prototype filter')
%
%Prototype filter H(z)=z^(-34)-T(z)
%
h=-t;h(35)=1+h(35);
figure(7)
[H,W]=zeroam(h,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('New ECG-Filter: Prototype H(z)=z^(-34)-T(z)')
figure(8)
impz(h)
xlabel('n in samples');
ylabel('Impulse response');
title('New ECG-Filter: Prototype H(z)=z^(-34)-T(z)')
%%%%%%%%%%%%%%%%%%%%%%%%%
%Then H(z^4)
%%%%%%%%%%%%%%%%%%%%%%%%%
for k=1:69 hh(4*k-3)=h(k);end
figure(9)
[H,W]=zeroam(hh,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('New ECG-Filter: H(z^4)')
figure(10)
```

```
impz(hh)
xlabel('n in samples');
ylabel('Impulse response');
title('New ECG-Filter: H(z^4)')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Optimized ECG Prototype Filter: H(z)=z^(-M)-T(z)
%T(z)=F(z^3)G(z), M=half the order of T(z)
%F(z) and G(z) have been designed using the
%synthesis technique described in
%T. Saramaki, "Finite impulse response Filter
%Design" in Handbook for Digital Signal Processing,
%S. K. Mitra and J. F. Kaiser, Eds, John Wiley &
%Sons, 1993,  pp. 241-245
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
clear all;
%Design F(z)
%
load ber11;
ff=rot90(ber11);
%
%Design G(z)
%
load ber12;
gg=rot90(ber12);
%
%Impulse response of T(z)=F(z^3)G(z)
%
for k=1:17 fff(3*k-2)=ff(k);end
t=conv(fff,gg);
[H,W]=zeroam(fff,.0,1.,2000);
[HH,W]=zeroam(gg,.0,1.,2000);
figure(11)
plot(100*W/pi,20*log10(abs(H)),100*W/pi,20*log10(abs(HH)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('Components for T(z)=F(z^3)G(z): F(z) and G(z) of orders 16 and 6')
figure(12)
[H,W]=zeroam(t,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('Optimized ECG-Filter: T(z): complementary for the prototype filter')
%
%Prototype filter H(z)=z^(-27)-T(z)
%
h=-t;h(28)=1+h(28);
figure(13)
[H,W]=zeroam(h,.0,1.,2000);
plot(100*W/pi,20*log10(abs(H)));
axis([0 100 -120 10]);grid;
ylabel('Amplitude in dB');
xlabel('Frequency in Hz');
title('Optimized ECG-Filter: Prototype H(z)=z^(-27)-T(z)')
figure(14)
impz(h)
xlabel('n in samples');
ylabel('Impulse response');
```