DESIGN OF EFFICIENT FIR FILTERS BASED ON MULTIRATE AND COMPLEMENTARY FILTERING

Tapio Saramäki and Tor Ramstad

One of the best approaches to significantly reducing the number of multiplications and additions required in implementing FIR filters is to use multirate filtering. It is very straightforward to apply this technique to designing filters whose passband bandwidth is relatively narrow or relatively wide [CR75], [RA75], [MI78a], [MI78b], [SA84a], [CH83], [CH84], [SA86]. However, for narrow transition-band filters with arbitrary passband bandwidth, conventional multirate filtering techniques cannot be used in a straightforward manner. In this section, we consider a general multirate filtering method [RA88], [RA90] which can be used for designing filters with any passband bandwidth. This method is an extension of the Jing-Fam method [JI84] (see Section 4-11-5 in [SA93]) in the sense that multirate techniques are included to reduce the computation complexity even further. Only low-pass and high-pass filter designs are treated, although the proposed technique is easily adapted to bandpass and bandstop filters as well.

1 Basic Building Blocks

In the general multirate filtering approach proposed in [RA88], [RA90], the first step is to reduce the multiplication and addition rates by replacing a filter $H(z)$ shown in Fig. 1(a) by a cascade of a two-port and a termination filter $H_1(z)$ as shown in Fig. 1(b). The two-port and the termination filter are determined such that the resulting overall system meets the same specifications as $H(z)$. Then, $H_1(z)$ is synthesized in a similar manner as a cascade of another two-port and a new termination filter $H_2(z)$, resulting in an overall system shown in Fig. 1(c). The purpose of each step is to gradually reduce the overall multiplication and addition rates. The process is repeated until the processing left in the remaining termination $H_i(z)$ is negligible or the total processing is minimized. Only two basic building blocks shown in Fig. 2 are needed. Before explaining how to use these building blocks, we distinguish between two filter classes: narrow-band filters and broad-band filters. The narrow-band filter is defined as a filter where the passband occupies a bandwidth less than $f_s/4$ where f_s is the sampling frequency. If the passband occupies more bandwidth than $f_s/4$, then we denote this a broad-band filter.

1-1 Design of Narrow-Band Filters

If the overall filter $H(z)$ is narrowband, then it is well-known from the theory of multirate filtering that the multiplication rate can be reduced by using the building block of Fig. 2(a) as a two-port in Fig. 1(b). The upper part of this block decreases the sampling rate by a factor of $r = M/N$, whereas the lower part increases the sampling rate by the same factor. As an example, Fig. 3(a) gives overall low-pass filter specifications and the requirements for the decimation and interpolation filters $H_D(z)$ and $H_I(z)$ for $r = 2$ ($N = 1$ and $M = 2$), whereas Fig. 3(b) gives the resulting specifications for $H_1(z)$. The sampling rate of the filter $H_1(z)$ is lowered by $r = M/N$ as compared to $H(z)$ and the relative transition bandwidth is increased by the same factor. Since length of an FIR filter is (see Section 4-8-2 in [SA93]) roughly inversely proportional to the relative transition bandwidth, we conclude that the processing in $H_1(z)$ is lowered by a factor of $(M/N)^2$ as compared to $H(z)$. A narrow-band high-pass filter can be synthesized in the same manner, as we shall see later on. By selecting $r = M/N$ in Fig. 2(a) such that it leaves the termination filter $H_1(z)$ as a broad-band filter, this two-port can be used to convert the design problem from a narrow-band filter to a broad-band filter. In order to be able to repeat the synthesis procedure, the remaining problem is to find a two-port converting the design from a broad-band filter to a narrow-band filter.

1-2 Design of Broad-Band Filters

If a broad-band filter is to be designed, we try to transform this into an equivalent structure containing a narrow-band filter in order to easily apply decimation/interpolation techniques. If $H(z)$ represents a broad-band linear phase filter, we can construct an equivalent problem in terms of a narrow-band complementary highpass filter $H_1(z)$ as (cf. Section 4-8-3 in [SA93])

$$H(z) = z^{-D} - H_1(z) \tag{1}$$

where z^{-D} is the delay of the two filters, which are both Type I filters of order $2D$. If the impulse response coefficients of $H_1(z)$ are $h_1[n]$ then the zero-phase frequency responses of $H(z)$ and $H_1(z)$ can be written as

$$H_1(f/f_s) = h_1[D] + \sum_{n=1}^{D} 2h_1[D-n]\cos(2\pi nf/f_s) \tag{2a}$$

$$H(f/f_s) = 1 - H_1(f/f_s). \tag{2b}$$

According to Eq. (2b), the sum of these two zero-phase responses is unity. Therefore, these filters are called *complementary* filters. If it desired to design a broad-band low-pass filter $H(z)$ with the zero-phase response approximating unity on $[0, f_p]$ with ripple δ_p and zero on $[f_{st}, f_s/2]$ with ripple δ_s, then the synthesis is converted to the design of a narrow-band high-pass filter $H_1(z)$ whose zero-phase response approximates zero on $[0, f_p]$ with ripple δ_p and unity on $[f_{st}, f_s/2]$ with ripple δ_s, as shown in Fig. 4. $H(z)$ can be implemented in terms of the delay term z^{-D} and and a narrow-band filter $H_1(z)$ using the building block of Fig. 2(b) in Fig. 1(b). The building block of Fig. 2(b) provides thus the desired broadband-to-narrowband transformation which is opposite to that of the block of Fig. 2(a). Since $H_1(z)$ is a narrow-band high-pass filter, we can implement it using a building block of Fig. 2(a) cascaded with a broad-band filter $H_2(z)$. The resulting overall filter is then as shown in Fig. 1(c), where the first and second two-ports use the building blocks of Figs. 2(b) and 2(a), respectively.

1-3 Overall structrure

Using the above fundamental bulding blocks, $H_1(z)$ in Fig. 1(b) or $H_2(z)$ in Fig. 1(c) can be realized using the two-ports of Fig. 2 and a new termination filter. Figure 5 shows the resulting overall multirate multistage realization, where the remaining termination filter is denoted by $\widehat{H}(z)$. In this figure, (a_i, b_i) is either $(1, 0)$ or $(-1, 1)$. In the former case, the additional delay term z^{-D_i} is not used. It has turned out that, after studing various different sub-structures [RA88], [RA90], that only two decimation/interpolation two-ports are needed, one using decimation and interpolation by $r = 2$ ($N = 1$, $M = 2$) and the other decimation and interpolation by $r = 2/3$ ($N = 3$, $M = 2$). In both of these cases, the decimation and interpolation filters $H_D(z)$ and $H_I(z)$ can be efficiently constructed using half-band filters, which are considered in the next subsection.

2 Half-Band Filters

For constructing the decimation filter $H_D(z)$ and the interpolation filter $H_I(z)$, half-band filters (see Section 4-10-3 in [SA93]) are particularly efficient. The transfer function of these filters can be written as

$$H_D(z) = \frac{1}{2}z^{-K} + G(z^2), \tag{3}$$

where K is an odd integer and the order of $G(z^2)$ is K in z^2 or $2K$ in z. When used for decimation or interpolation, this filter can be implemented using the commutative structures [CR83] shown in Fig. 6. The delay branch $z^{-(K-1)/2}$ can be shared with $G(z)$. In the decimation case, this can be done using the transposed direct-form realization exploiting the coefficient symmetry of $G(z)$. In the interpolation case, the direct-from realization is used. When the symmetry in the coefficients of $G(z)$ is exploited, we need only $(K+1)/2$ multipliers plus a trivial multiplication by 0.5 in the decimation case. Since $G(z)$ is working in both cases at the lower sampling rate of $\widehat{f}_s = f_s/2$, the simultaneous implementation of both $H_D(z)$ and $H_I(z)$ requires only $f_s(K+1)/2$ multiplications per second. The half-band filters are characterized by the facts that their passband and stopband ripples are equal and the passband and stopband edges are related via

$$f_{st} = f_s/2 - f_p. \tag{4}$$

3 Design Equations

This subsection gives the basic design equations for synthesizing the proposed filters. All what is needed is to determine the conditions under which a filter $H(z)$ can be synthesized in terms of the building blocks of Subsection 2 and a termination filter $\widehat{H}(z)$. After knowing these conditions, we are able to repeat the overall synthesis procedure.

Let $H(z)$ be a low-pass filter with passband and stopband edges of $f_c \pm \Delta$ and sampling rate of f_s. There are the following three cases which require different constructions for $H(z)$:

Case A: $f_c < f_s/4$ and $f_c + \Delta$ is not close to $f_s/4$.

Case B: $f_c > f_s/4$ and $f_c - \Delta$ is not close to $f_s/4$.

Case C: $f_s/4$ is close to or inside the interval $[f_c - \Delta, f_c + \Delta]$.

In *Case A*, $H(z)$ is a narrow-band low-pass filter and we use the building block of Fig. 2(a) with $N = 1$ and $M = 2$. When this block is cascaded with the termination $\widehat{H}(z)$, the relation between the z-transforms of the input signal $x(n)$ and the output signal $y(n)$ becomes

$$Y(z) = F_1(z)X(z) + F_2(z)X(-z), \tag{5a}$$

where

$$F_1(z) = \widehat{H}(z^2)H_D(z)H_I(z) \tag{5b}$$

and

$$F_2(z) = \widehat{H}(z^2)H_D(-z)H_I(z). \tag{5c}$$

Here, $F_1(z)$ is a conventional transfer function from the input to the output. This transfer function must satisfy the conditions stated for $H(z)$. $F_2(z)X(-z)$ is the aliased term due to the sampling rate alteration and the response of $F_2(z)$ must be small in the frequency range $[0, f_s/2]$. The desired result is obtained by selecting the edges of $\widehat{H}(z)$ working at the sampling rate of

$$\widehat{f_s} = f_s/2 \tag{6}$$

to be those of the overall filter, i.e., $\widehat{f_c} \pm \Delta$ with

$$\widehat{f_c} = f_c. \tag{7}$$

Because of the periodicity of $\widehat{H}(z^2)$, it has an extra passband $[f_s/2-(f_c-\Delta), f_s/2]$ and an extra transition band $[f_s/2-(f_c+\Delta), f_s/2-(f_c-\Delta)]$ (see Fig. 7). The required passband and stopband edges for $H_D(z)$ and $H_I(z)$ are

$$f_p^{(D)} = f_c - \Delta, \quad f_{\mathrm{st}} = f_s/2 - (f_c + \Delta). \tag{8}$$

The resulting $H_D(z)$ and $H_I(z)$ preserve the first passband region of $\widehat{H}(z^2)$ and attenuate the extra transition band and the extra passband of $\widehat{H}(z^2)$, giving the desired response for $F_1(z)$. The passband ripple is at most the sum of the ripples of the three filters. The stopband ripple is in the stopband region of $\widehat{H}(z^2)$ at most the stopband ripple of $\widehat{H}(z^2)$ and in the extra transition band and in the extra passband of $\widehat{H}(z^2)$ at most the product of the stopband ripples of $H_D(z)$ and $H_I(z)$. In the case of $F_2(z)$, $H_I(z)$ attenuates the second transition band and the second passband of $\widehat{H}(z^2)$ and $H_D(-z)$ takes care of the lower ones, resulting in a small aliased term $F_2(z)X(-z)$. The maximum value of the aliased components is limited by the maximum of the stopband ripples of the three filters. It has been observed experimentally that the required stopband edge of $H_D(z)$ and $H_I(z)$ can be increased to

$$f_{\mathrm{st}}^{(D)} = f_s/2 - (f_c + \Delta/3). \tag{9}$$

This is because the termination filter $\widehat{H}(z^2)$ provides some attenuation in the transition bands near the stopband edges. Using half-band filters with properties as discussed above for $H_D(z)$ and $H_I(z)$, the passband edge has to be selected as

$$f_p^{(D)} = f_c + \Delta/3 \tag{10}$$

to give the desired stopband edge as given by Eq. (9).

In *Case B*, $H(z)$ is a broad-band design and we first use the building block of Fig. 2(b) to convert the problem to the design of narrow-band high-pass filter. The passband edge is $f_c+\Delta$ and the stopband edge is $f_c-\Delta$. This filter can be constructed as in Case A using the building block of Fig. 2(a) with $N = 1$ and $M = 2$. The input-output relation for the overall filter consiting of the two building blocks and the termination filter becomes

$$Y(z) = [z^{-D} - F_1(z)]X(z) - F_2(z)X(-z), \tag{11}$$

where $F_1(z)$ and $F_2(z)$ are given by Eqs. (5b) and (5c). In this case, the edges of $\widehat{H}(z)$ working at the sampling rate of $\widehat{f_s} = f_s/2$ are selected to be $\widehat{f_c} \pm \Delta$, where

$$\widehat{f_c} = f_s/2 - f_c. \tag{12}$$

$H_D(z)$ and $H_I(z)$ are high-pass filters with edges

$$f_{\rm st}^{(D)} = f_{\rm s}/2 - f_c + \Delta, \quad f_p = f_c + \Delta. \tag{13}$$

The resulting $\widehat{H}(z^2)$ has two passband regions $[0, f_{\rm s}/2 - (f_c + \Delta)]$ and $[f_c + \Delta, f_{\rm s}/2]$ and a stopband region $[f_{\rm s}/2 - (f_c - \Delta), f_c - \Delta)]$ (see Fig. 8). $H_D(z)$ and $H_I(z)$ preserve the second passband region and attenuate the first passband and first transition band. The resulting $F_1(z)$ is then the desired narrow-band high-pass filter, and the complementary filter $z^{-D} - F_1(z)$ is the desired broad-band low-pass filter. The passband ripple of $z^{-D} - F_1(z)$ is at most the product of the stopband ripples of $H_D(z)$ and $H_I(z)$ in the first passband and in the first transition band of $\widehat{H}(z^2)$ and at most the stopband ripple of of $\widehat{H}(z^2)$ in the stopband of $\widehat{H}(z^2)$. The stopband ripple is at most the sum of the passband ripples of the three filters. In the case of $F_2(z)$, $H_D(-z)$ ($H_I(z)$) attenuates the second (first) passband and transition band regions and the maximum value of the aliased components is the maximum of the stopband ripples of the three filters.

Because of the effect of $\widehat{H}(z^2)$, the stopband edge of $H_D(z)$ and $H_I(z)$ can be reduced to

$$f_{\rm st}^{(D)} = f_{\rm s} - f_c + \Delta/3. \tag{14}$$

When half-band filters are used, then this is the required stopband edge.

In *Case C*, $f_{\rm s}/4$ is either in the transition band of $H(z)$ or the passband or the stopband edge of $H(z)$ is close to $f_{\rm s}/4$. In the former case, we cannot use $N = 1$ and $M = 2$ at all. In the latter case, the transition bands of $H_D(z)$ and $H_I(z)$ become narrow, resulting in high filter orders. To avoid this problem, one alternative is to use the building block of Fig. 2(a) with $N = 3$ and $M = 2$. In this case, the input-output relation is

$$V(z) = F_1(z)X(z^3) + F_2(z)X(-z^3) \tag{15a}$$

$$Y(z) = V(z^{1/3}) + V(z^{1/3}e^{j2\pi/3}) + V(z^{1/3}e^{j4\pi/3}), \tag{15b}$$

where $F_1(z)$ and $F_2(z)$ are given by Eqs. (5b) and (5c), respectively. The relation between the input of $H_D(z)$ and the output of $H_I(z)$ is the same as in Case A. The basic difference is that the input sampling rate of $H_D(z)$ is now $f_{\rm s}' = 3f_{\rm s}$ and the input of $H_D(z)$ contains one and a half periods of the original input signal spectrum [see Fig. 9(a)]. The specifications for the overall filter consisting of the termination filter, the decimator, and the interpolator are the same except that the sampling rate is now $f_{\rm s}' = 3f_{\rm s}$ [see Fig. 9(b)]. The second basic difference compared to Case A is that there are also aliased terms when finally decimating by three, as shown by Eq. (11b). However, because of filtering, the components aliased from the region $[f_{\rm s}/2, 3f_{\rm s}/2]$ are very small [see Fig. 9(c)].

The sampling frequency of the termination filter $\widehat{H}(z)$ is in this case $\widehat{f}_{\rm s} = 3/2f_{\rm s}$, i.e., 3/2 times that of the overall filter $H(z)$. The advantage of using $r = 2/3$ lies, however, in the following facts. First, the edges for the half-band designs of $H_D(z)$ and $H_I(z)$ are $f_p^{(D)} = f_c + \Delta/3$ and $f_{\rm st}^{(D)} = f_{\rm s}'/2 - (f_c + \Delta/3)$. Therefore, the relative transition bandwidth is very wide, resulting in very low filter orders. Second, the passband and stopband edges of $\widehat{H}(z)$ are close to $\widehat{f}_{\rm s}/3$ so that the design procedure can be easily repeated using the building block of Fig. 2(a) with $N = 1$ and $M = 2$.

It can be shown that by ignoring the aliased terms, the passband and stopband ripples of the overall design are at most

$$\delta_p = \sum_{k \in S} 2\delta^{(k)} + \widehat{\delta} \tag{16a}$$

$$\delta_s = \sum_{k \notin S} 2\delta^{(k)} + \widetilde{\delta}, \tag{16b}$$

where $\delta^{(k)}$ is the ripple of the kth decimator and interpolator and S is the set of stages in front of which there is an even number of building blocks of Fig. 2(b) or no blocks. $\widehat{\delta}$ ($\widetilde{\delta}$) is the passband (stopband) ripple of the termination filter if there is an even number of delay blocks in front of it. Otherwise, it is the stopband (passband) ripple. The simplest way to determine the required ripples is to make all the terms in the summations equal.

4 Design Examples

As an example we consider the design of a low-pass filter with passband and stopband edges of $0.2f_s$ and $0.201f_s$ [$(0.2005 \pm 0.0005)f_s$] and passband and stopband ripples of 0.001 (60-dB attenuation). The minimum order of a conventional direct-form design to meet these criteria is 3256, requiring 1629 multiplications per input sample. Using the synthesis procedure described in the previous sections, the first building block is selected to be the one in Fig. 2(a) with $N = 1$ and $M = 2$. The edges of the termination filter are, in terms of its sampling rate $f_s^{(1)} = f_s/2$, $(0.401 \pm 0.001)f_s^{(1)}$. The passband edge for both $H_D(z)$ and $H_I(z)$ is $(0.2005+0.0005/3)f_s$. The termination filter is now broadband. Therefore, we use a cascade of the building blocks of Figs. 2(b) and 2(a). The edges of the termination are $(0.198 \pm 0.002)f_s^{(2)}$ where $f_s^{(2)} = f_s/4$ is its sampling rate. $H_D(z)$ and $H_I(z)$ are high-pass half-band filters with stopband edge of $(0.099 + 0.001/3)f_s^{(1)}$. Proceeding in the same manner, the sampling rates for the third, fourth, fifth and sixth terminations become $f_s^{(k)} = f_s/2^k$ for $k = 3, 4, 5, 6$. For the third and fifth stages, we use the building block of Fig. 2(a) with $N = 1$ and $M = 2$ and the passband edges of the subfilters are $(0.198 + 0.002/3)f_s^{(2)}$ and $(0.208 + 0.008/3)f_s^{(4)}$, respectively. For the fourth and sixth stages, we use a cascade of the building blocks of Figs. 2(b) and 2(a) with $N = 1$ and $M = 2$. The subfilters are high-pass half-band filters with stopband edges of $(0.104 + 0.004/3)f_s^{(3)}$ and $(0.084 + 0.016/3)f_s^{(5)}$, respectively. If four or six stages are used, the edges of the remaining terminations are $(0.208 \pm 0.008)f_s^{(4)}$ and $(0.168 \pm 0.032)f_s^{(6)}$, respectively.

In the case of four stages, the ripples for all the stages become 0.0002 when determined from Eq. (16) in such a way that all the terms in the summations become equal. In the case of six stages, the ripples are 0.001/7.

With four stages the number of multipliers (the filter orders) for the decimation and interpolation stages are 11 (42), 4 (14), 11 (42), and 4 (14). The implementation of all the decimators and interpolators requires 16.25 multiplications per input sample. The minimum order of a direct-form FIR filter to meet the criteria of the termination is 264. This filter requires 133 multipliers. Since it is working at the sampling rate of $f_s/16$, it requires 8.3125 multiplications per input sample. The overall multiplication rate is thus 24.5625. For the first building block of Fig. 2(b), the delay term is z^{-1210} and, for the second block, z^{-278}. The number of delay elements required in implementing the overall filter is 1864, which is lower than that for the direct-form design (3256). The delay for the unaliased component is 2462, whereas the delay of the direct-form equivalent is 1628. The multiplication rate can be reduced by designing the termination in the form $F(z^2)G(z)$ (see Section 4-11-3 in [SA93]). The required orders of $F(z)$ and $G(z)$ are 136 and 12, respectively. In this case, the overall multiplication rate is 21.

When six stages are used, then the number of multipliers (orders) for the decimators and interpolators become 12 (46), 4 (14), 11 (42), 4 (14), 15 (58), and 4 (14). The order of the termination is 70. The overall multiplication rate is in this case 18.875. The price paid for the reduction in the multiplication rate is the increased overall delay (3970) and the increased number of delay elements (4700).

When four stages are used, then the overall output can be written in the form

$$Y(e^{j\omega}) = \sum_{k=0}^{15} H_k(e^{j\omega})X(e^{j(\omega-2k\pi/16)}),$$

so that there is an unaliased component and 15 aliased components. Figure 10(a) gives, in the case where the termination is synthesized in the form $F(z^2)G(z)$, $|H_0(e^{j\omega})|$ in dB (amplitude response for the unaliased component). Figure 10(b), in turn, gives $|H_6(e^{j\omega})|$ (solid line), $|H_8(e^{j\omega})|$ (dashed line), and $|H_4(e^{j\omega})|$ (dot-dashed line). The minimum attenuations for these responses are 74 dB, 74dB, and 83 dB, respectively. For the other responses, the minimum attenuation is more than 86 dB. All the aliased components are thus attenuated at least by 14 dB more than the required stopband attenuation of the filter, making them very small.

In order to examine how the multiplication rate varies as a function of the center of the transition band, Fig. 11 is provided. It plots a case where the width of the transition band is $0.001f_s$ and the passband and stopband ripples are 0.001, as in the previous example. In contructing this plot, the upper allowable number of stages has been fixed to be 8 and the upper limit for the overall delay has been two and a half times that of the direct-form equivalent (4070). For each center frequency, the number of stages has been selected to minimize the multiplication rate. In addition, it has been tested carefully when to use decimation and interpolation by $r = 3/2$. Usually, $r = 3/2$ gives the lowest multiplication rate when the center of the transition band for the termination is $0.225 - -0.275$ times the sampling rate. As seen from the figure, the maximum number of multiplications per input sample is in this case 40.

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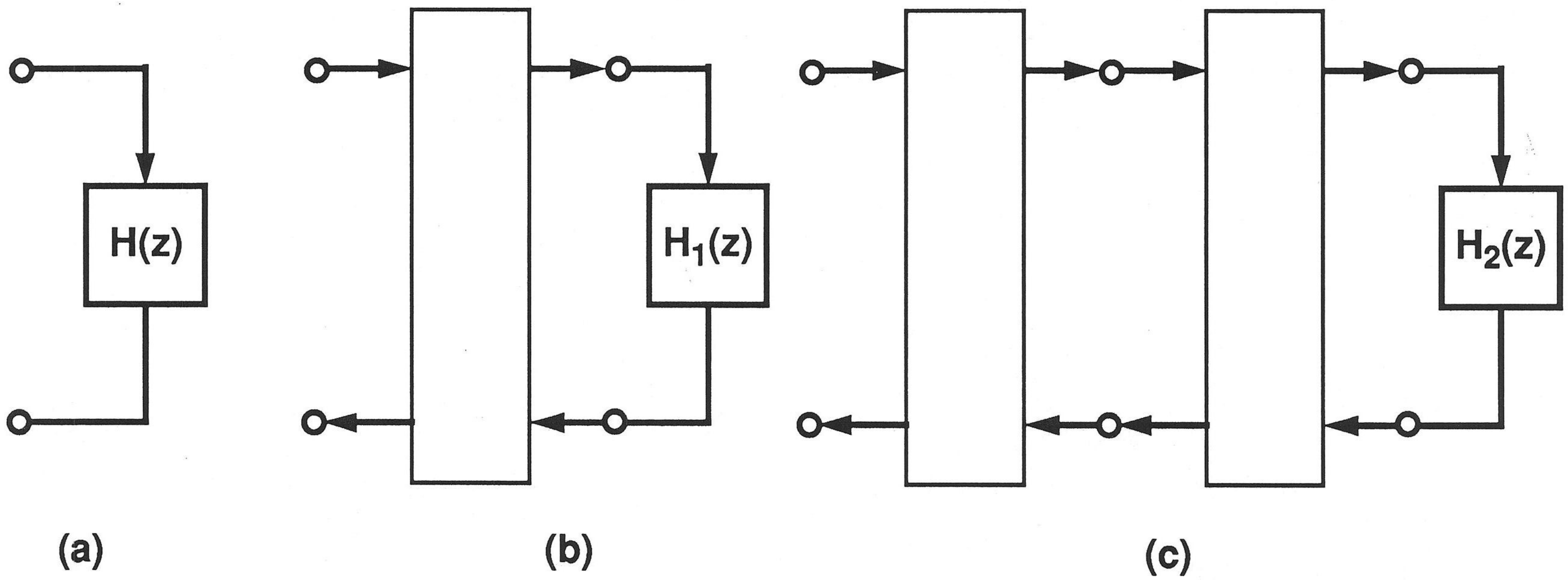


Fig. 1. *Implementation of an FIR filter using two-ports.*

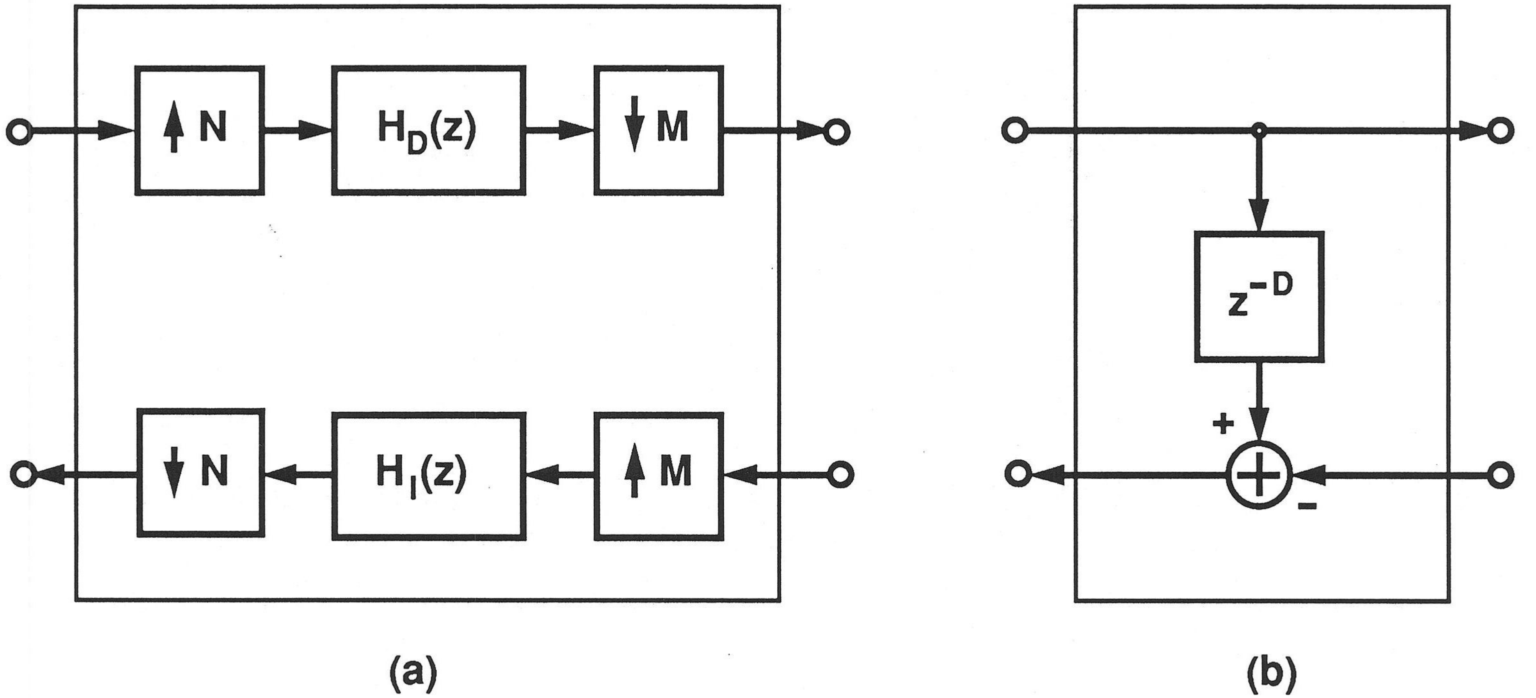


Fig. 2. *Two basic building blocks for synthesizing a computationally efficient multirate FIR filter.*

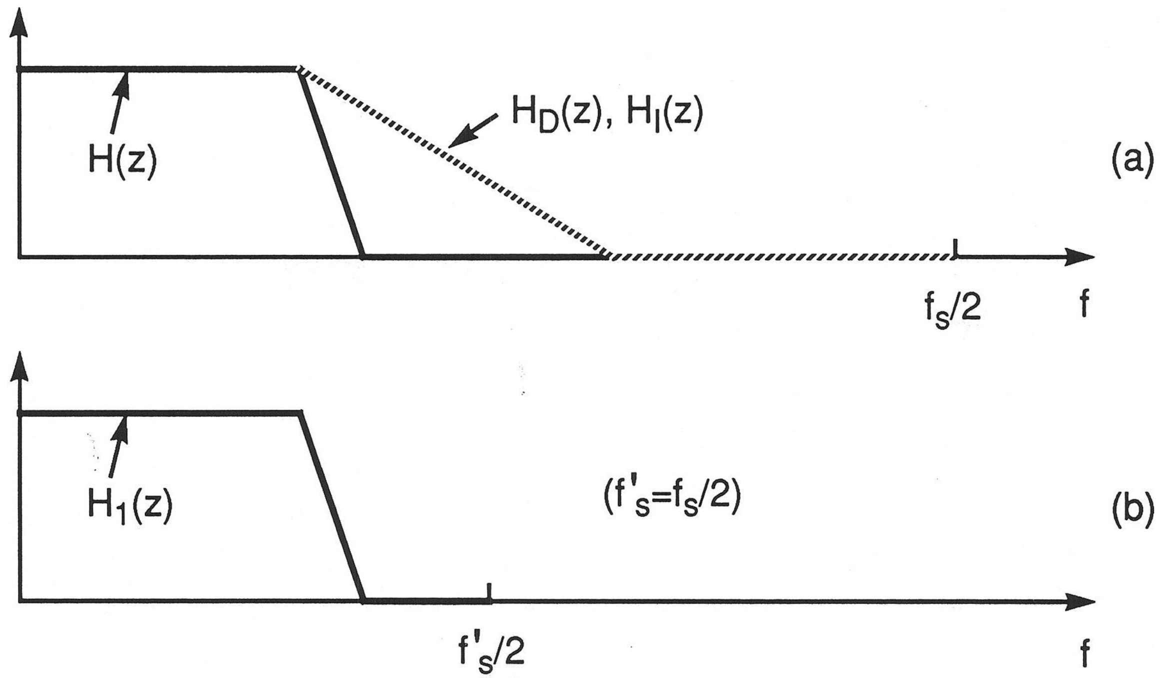
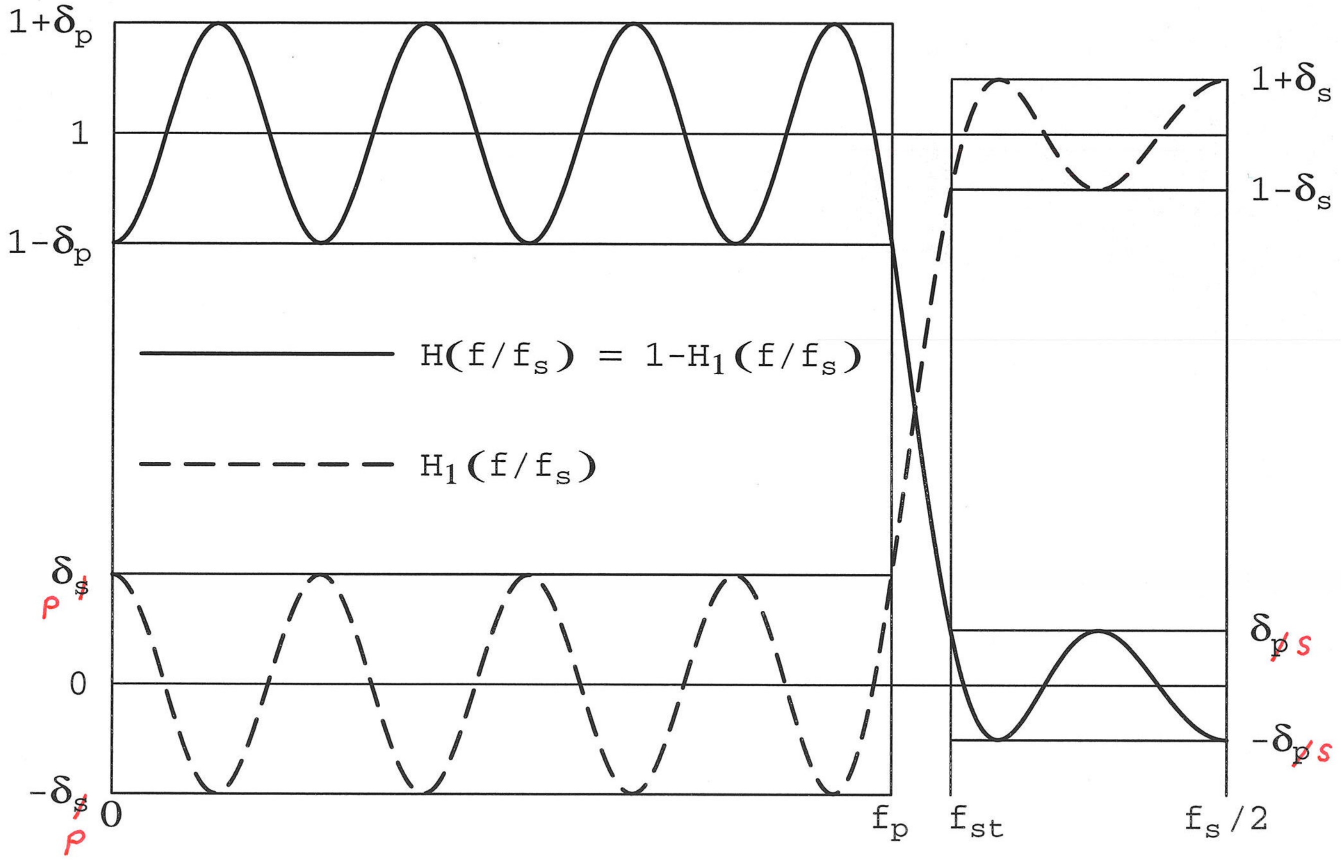


Fig. 3. *Design of a narrow-band filter $H(z)$ using the building block of Fig. 2(a) with $N = 1$ and $M = 2$ and a broad-band filter $H_1(z)$.*

Fig 4. *Frequency reponse of the filter $H_1(z)$ and the complementary response obtained using the system of Fig. 1(b) with the structure of Fig. 2(b) as a two-port.*



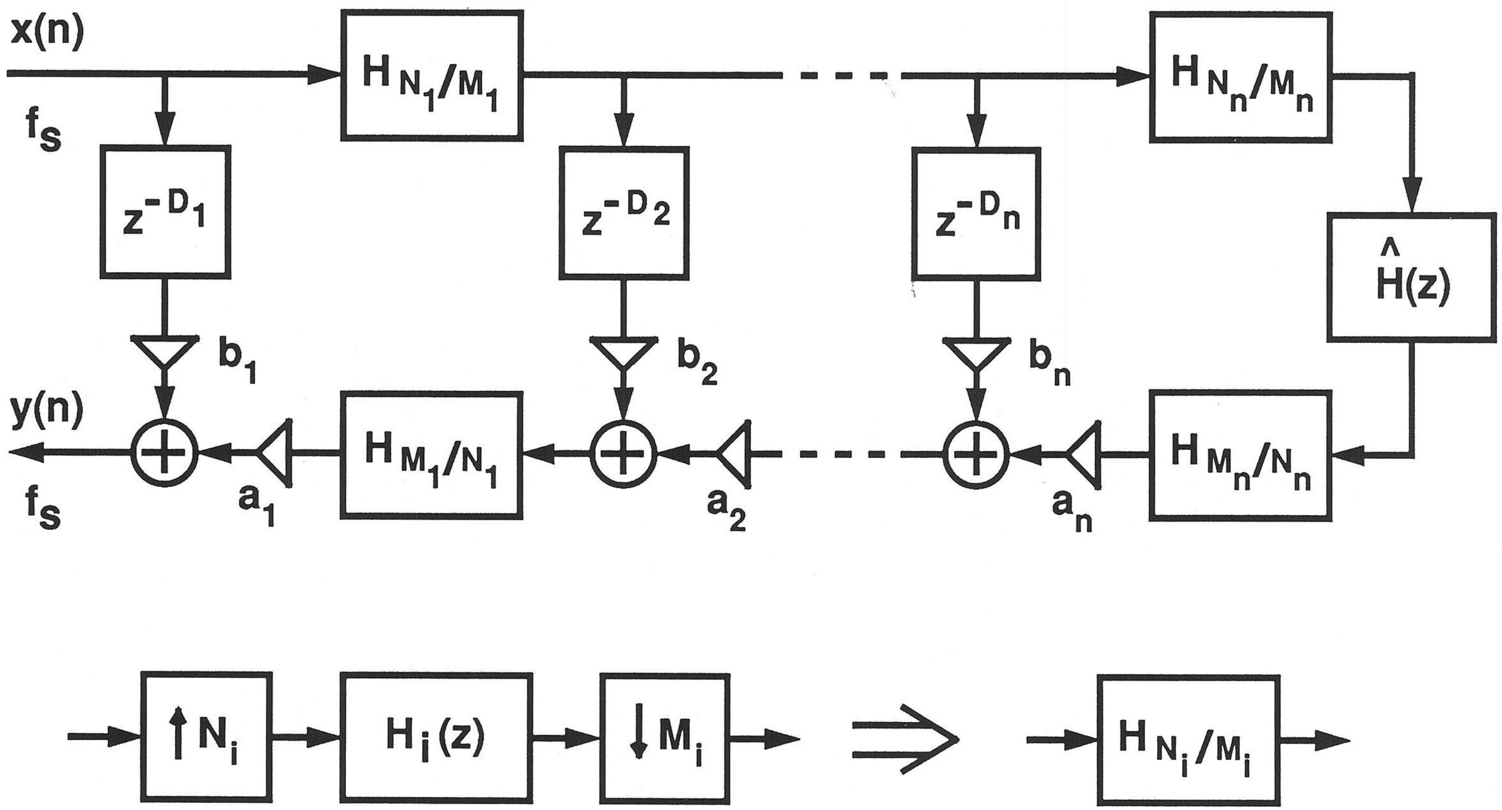


Fig 5. *Overall multistage multirate realization of an FIR filter.*

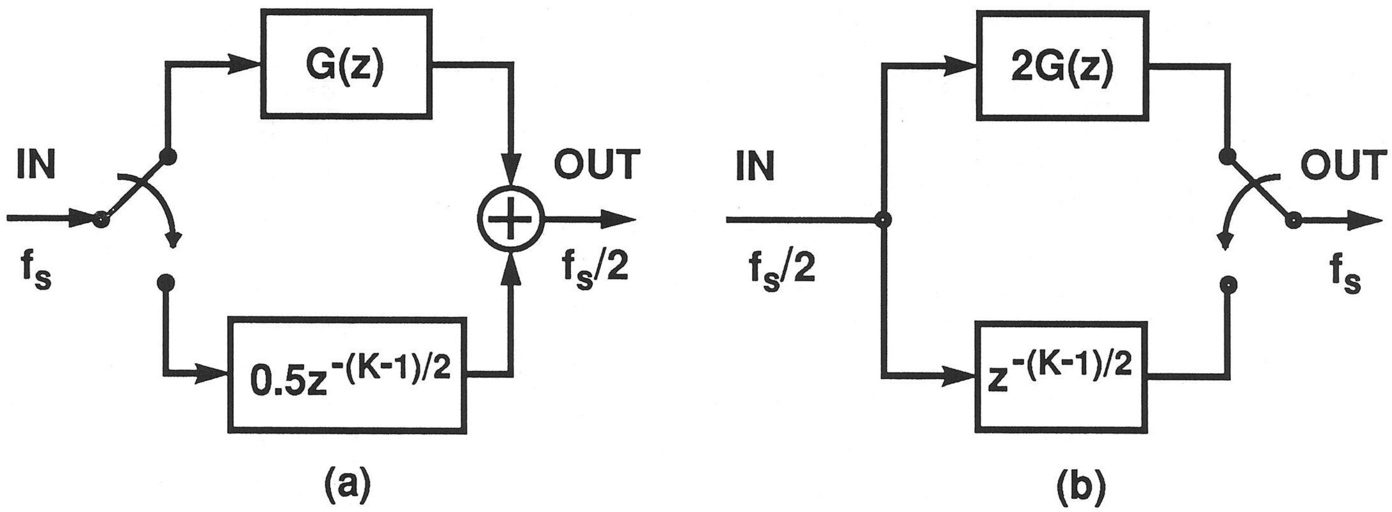


Fig. 6. *Commutative structures for FIR half-band filters. (a) Decimator. (b) Interpolator. Note that in the interpolation case the filter output must be multiplied by two to preserve the signal energy.*

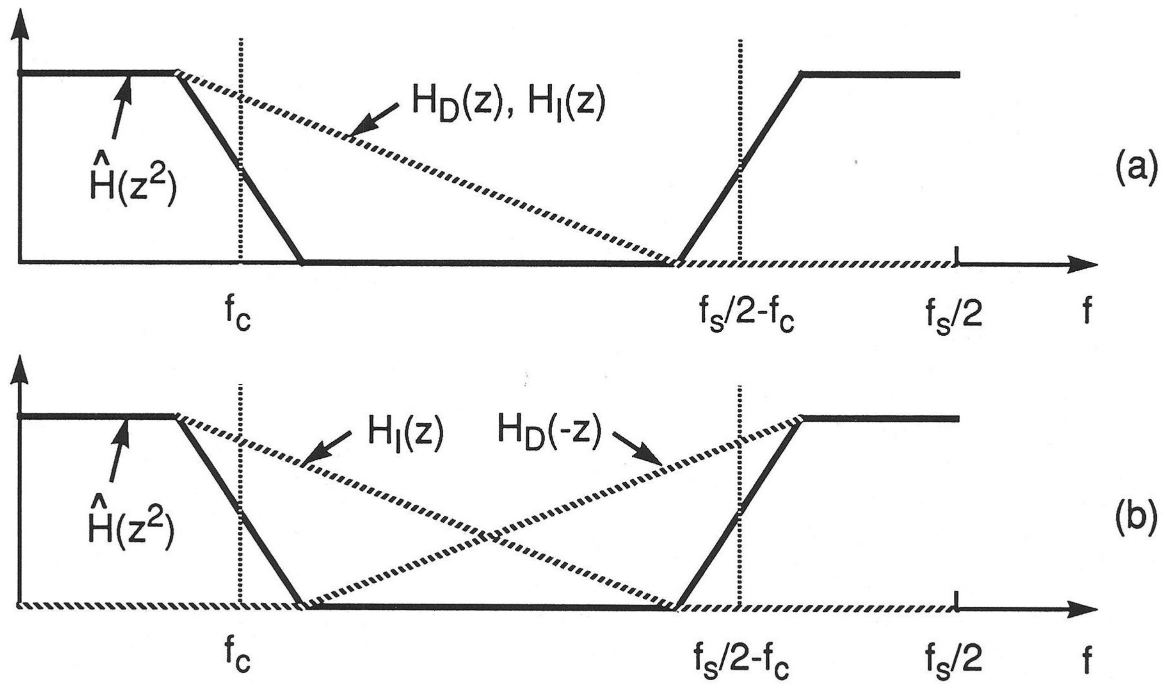


Fig. 7. *Transfer functions for the aliased and unaliased components in Case A. (a) Terms in* $F_1(z)$. *(b) Terms in* $F_2(z)$.

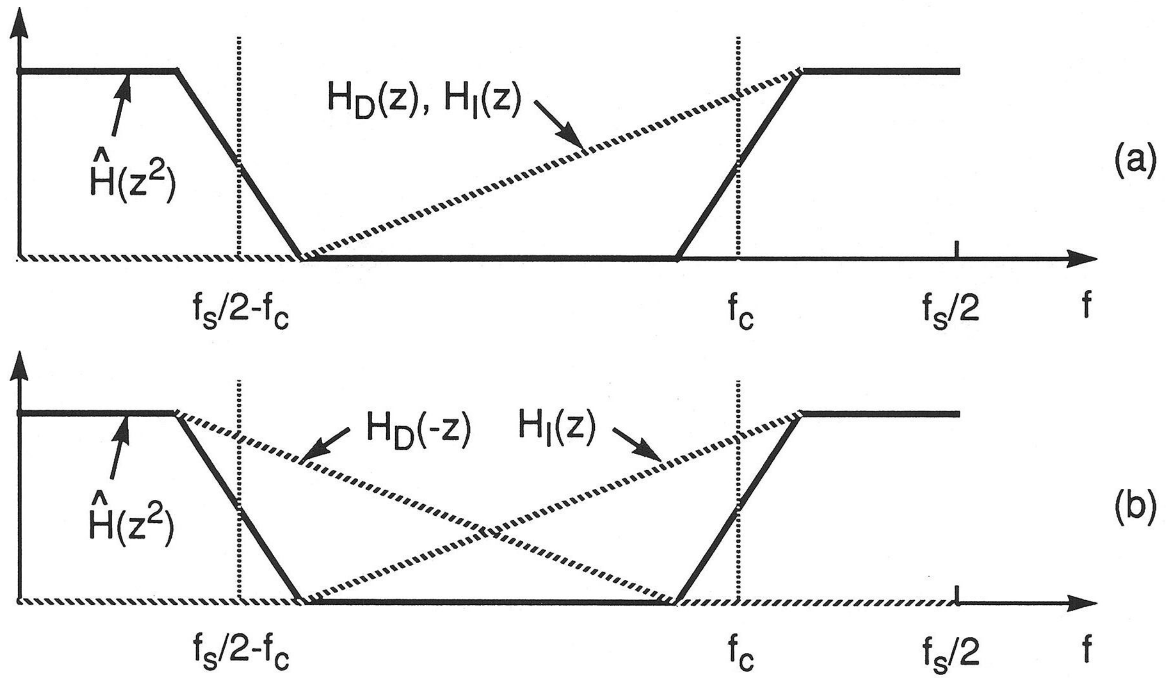
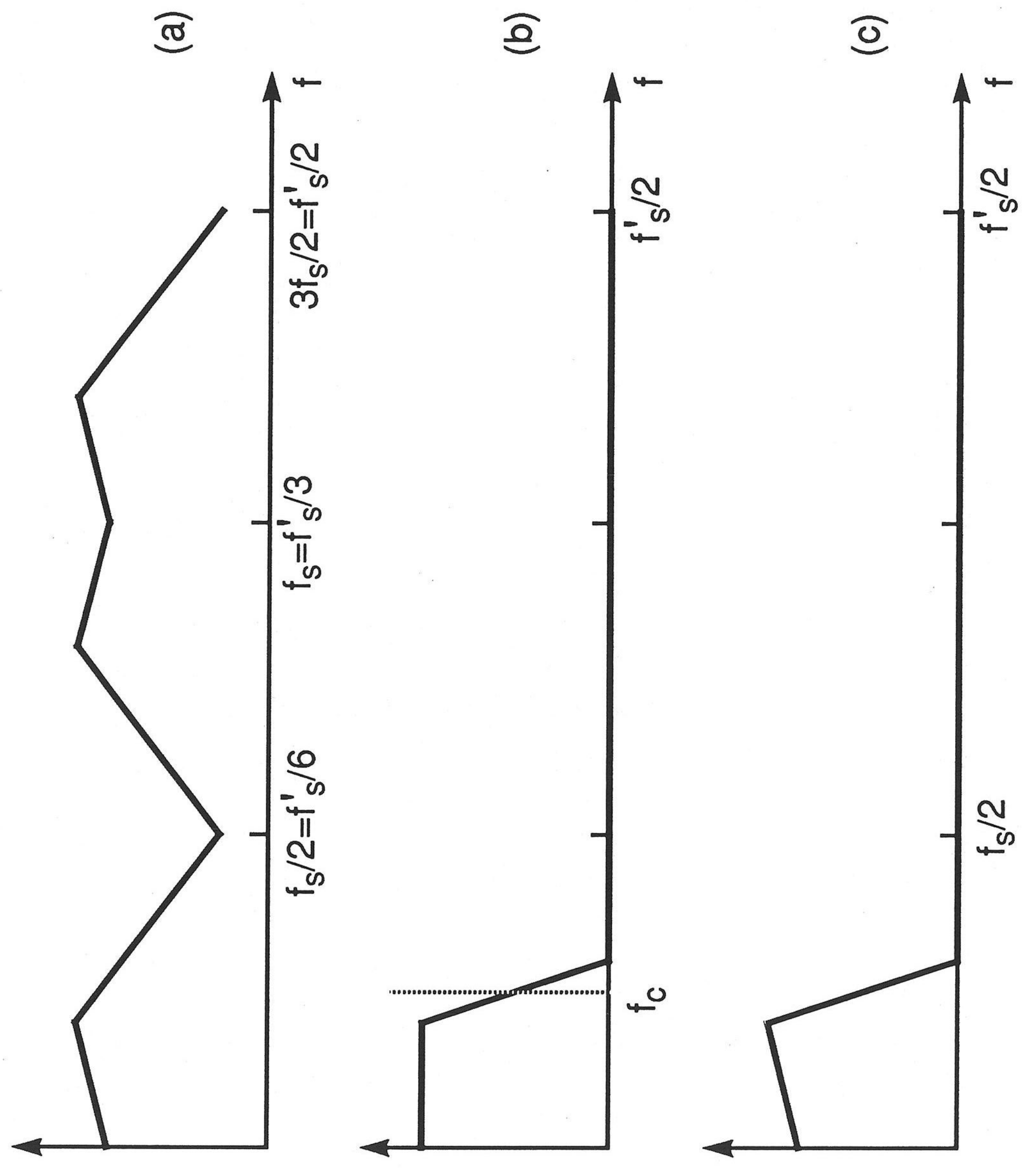
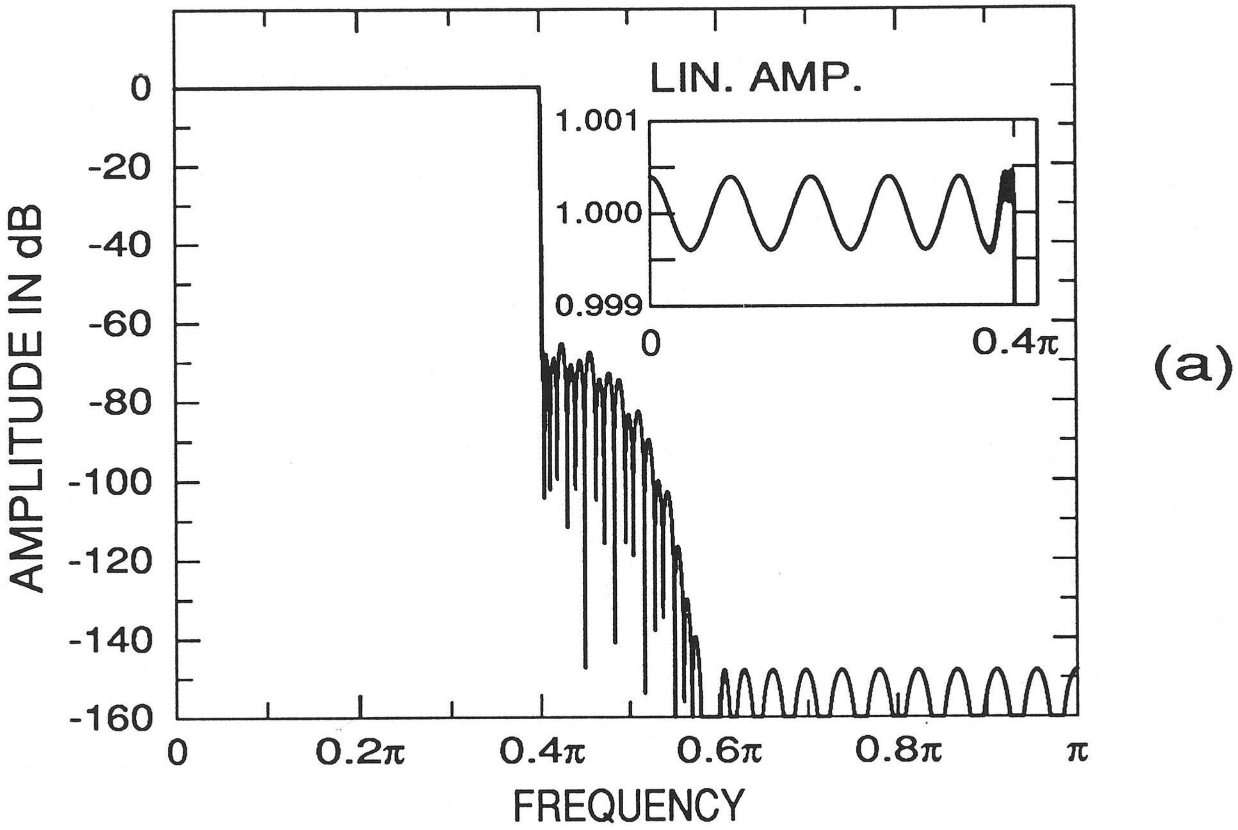


Fig. 8. *Transfer functions for the aliased and unaliased components in Case B. (a) Terms in $F_1(z)$. (b) Terms in $F_2(z)$.*

Fig. 9. *Filtering in Case C. (a) Periodic input signal spectrum of $H_D(z)$. (b) Transfer function from the input of $H_D(z)$ to the output of $H_I(z)$. (c) Output signal spectrum of $H_I(z)$ before sampling rate reduction by 3.*





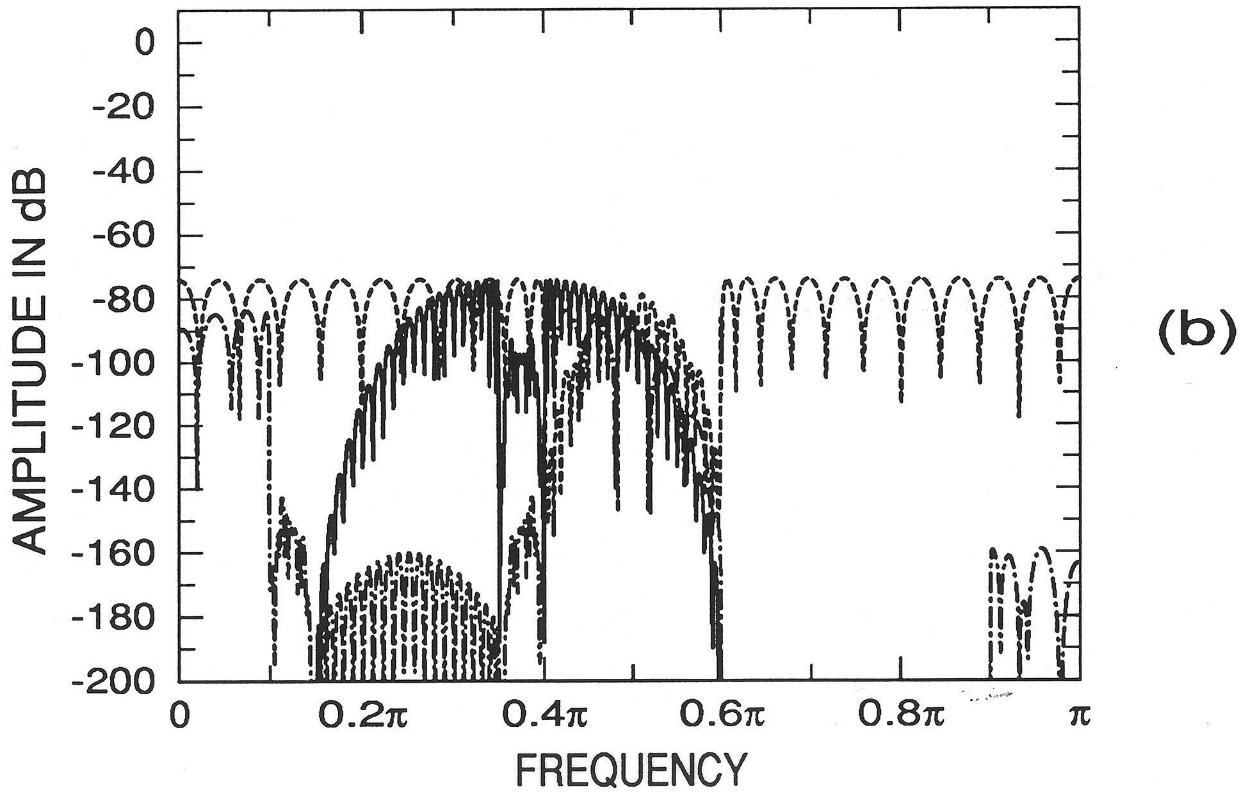


Fig. 10. *Responses for the proposed multirate filter with four decimation and interpolation stages. (a) Response for the unaliased component. (b) Responses for some aliased components.*

Fig. 11. *Multiplication rate versus the center of the transition band for filters with transition bandwidth of* $0.001 f_s$ *and passband and stopband ripples of 0.001.*