MULTIPLIER-FREE HALF-BAND FILTERS

- This pile of lecture notes shows how to design half-band FIR filters without general multipliers.
- These filters can be used as building blocks for constructing multiplier-free superresolution decimators and interpolators.
- They have been used in the article (a copy of this article as well as the conference talk are included):
- T. Saramäki, T. Karema, T. Ritoniemi, and H. Tenhunen, "Multiplier-free decimator algorithms for superresolution oversampled converters," in *Proc.* 1990 *IEEE International Symposium on Circuits and Systems* (New Orleans, Louisiana), pp. 3275–3278, May 1990.

What Are Half-Band FIR Filters?

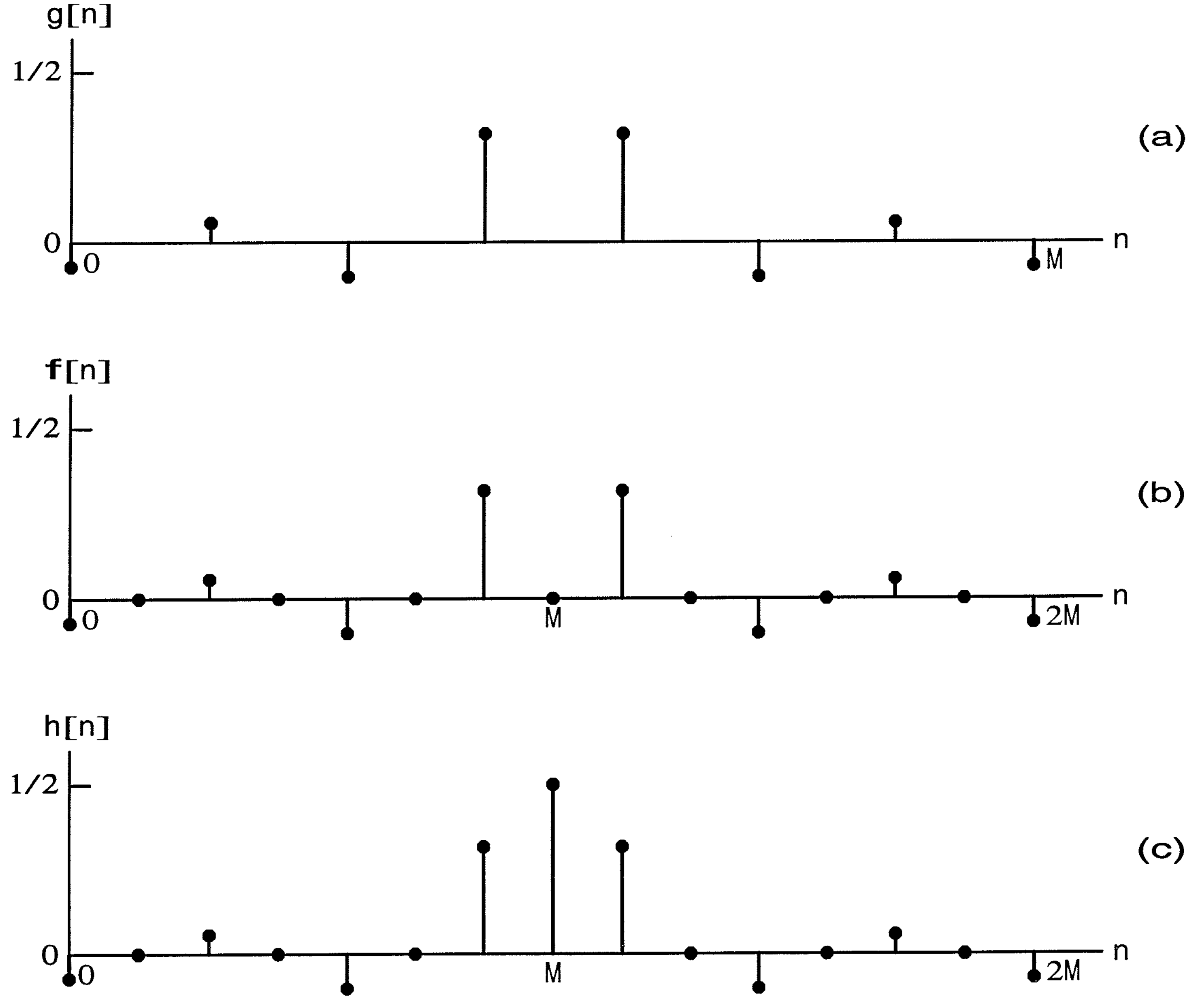
- For a half-band FIR filter, the transfer function is of the form
$$H(z) = \sum_{n=0}^{2M} h[n] z^{-n}, \quad h[2M-n] = h[n],$$
where M is odd.

- For these filters,
$$h[M] = 1/2$$
$$h[M+2r] = 0 \quad \text{for} \quad r = \pm 1, \pm 2, \ldots, \pm (M-1)/2.$$

- A filter satisfying these conditions can be generated in two steps by starting with a Type II (M is odd) transfer function
$$G(z) = \sum_{n=0}^{M} g[n] z^{-n}, \quad g[n] = g[M-n].$$

- In the first step, zero-valued impulse-response values are inserted between the $g[n]$'s [see Figures (a) and (b) in the following transparency], giving the following Type I transfer function of order $2M$:
$$F(z) = \sum_{n=0}^{2M} f[n] z^{-n} = G(z^2) = \sum_{n=0}^{M} g[n] z^{-2n}.$$

Generation of the Impulse Response of a Half-Band Filter



- The second step is then to replace the zero-valued impulse-response value at $n = M$ by $1/2$ [see Figure (c) in the previous transprency], resulting in the desired transfer function

$$H(z) = \sum_{n=0}^{2M} h[n]z^{-n} = \frac{1}{2}z^{-M} + F(z) = \frac{1}{2}z^{-M} + \sum_{n=0}^{M} g[n]z^{-2n}.$$

- This gives $h[M] = 1/2$, $h[n] = g[n/2]$ for n even, and $h[n] = 0$ for n odd and $n \neq M$, as is desired.

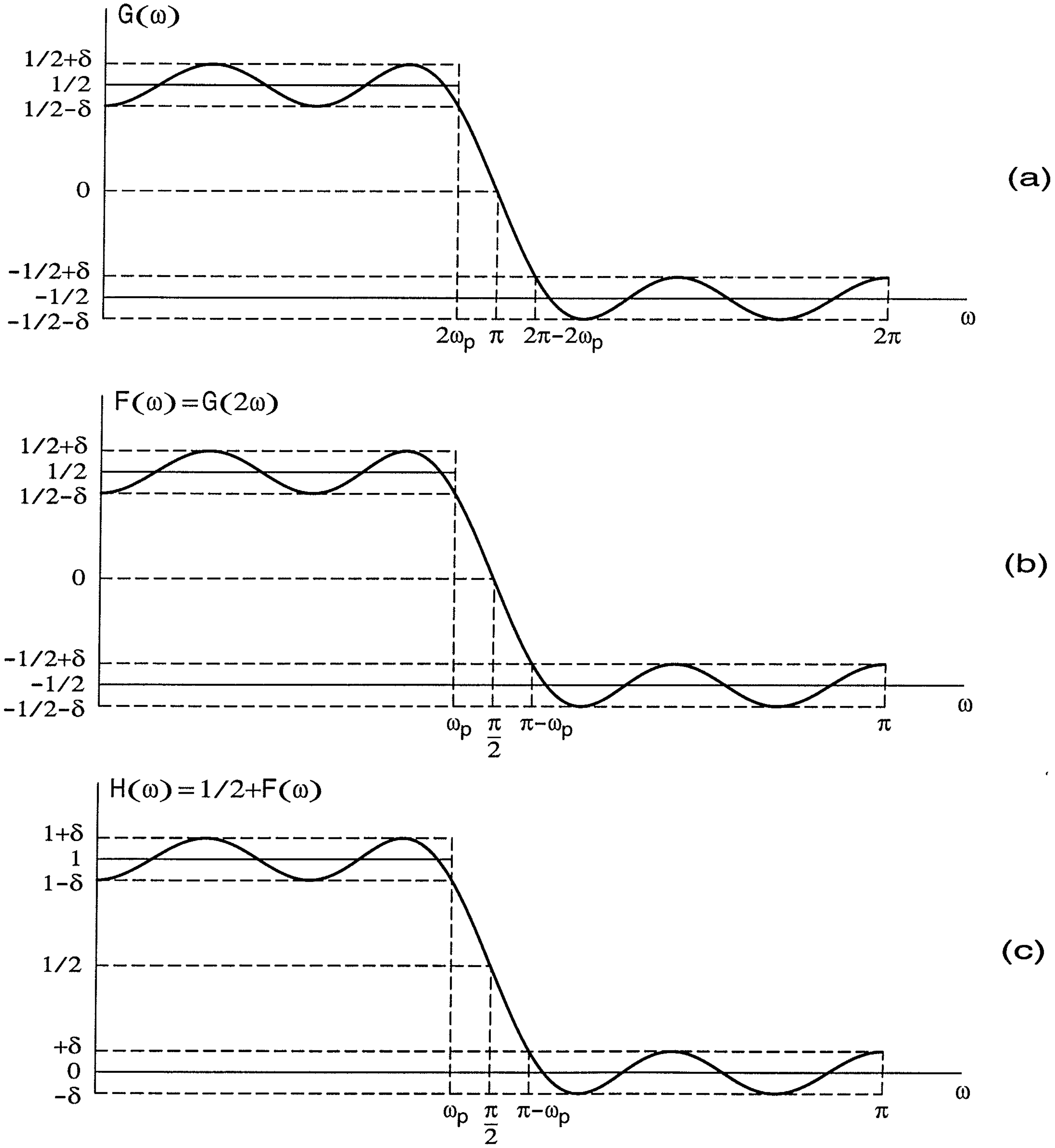
Filter Design

- The zero-phase frequency responses of $H(z)$, $F(z)$, and $G(z)$ are related through

$$H(\omega) = 1/2 + F(\omega) = 1/2 + G(2\omega).$$

- Based on these relations, the design of a low-pass half-band filter with passband edge at ω_p and passband ripple of δ can be accomplished by determining $G(z)$ such that $G(\omega)$ oscillates within $1/2 \pm \delta$ on $[0, 2\omega_p]$ [see Figure (a) in the following transprency].
- Since $G(z)$ is a Type II transfer function, it has one fixed zero at $z = -1$ $(\omega = \pi)$.
- $G(z)$ can be designed directly with the aid of the Remez algorithm using only one band $[0, 2\omega_p]$, $D(\omega) = 1/2$, and $W(\omega) = 1$.
- Since $G(z)$ has a single zero at $z = -1$, $G(\omega)$ is odd about $\omega = \pi$.
- Hence, $G(2\pi - \omega) = -G(\omega)$ and $G(\omega)$ oscillates within $-1/2 \pm \delta$ on $[2\pi - 2\omega_p, 2\pi]$.

Design of A Lowpass Half-Band Filter



- The corresponding $F(\omega) = G(2\omega)$ stays within $1/2 \pm \delta$ on $[0, \omega_p]$ and within $-1/2 \pm \delta$ on $[\pi - \omega_p, \pi]$ [see Figure (b) in the previous transparency].
- Finally, $H(\omega)$ approximates unity on $[0, \omega_p]$ with tolerance δ and zero on $[\pi - \omega_p, \pi]$ with the same tolerance δ [see Figure (c) in the previous transparency].
- For the resulting $H(\omega)$, the passband and stopband ripples are thus the same and the passband and stopband edges are related through $\omega_s = \pi - \omega_p$.
- In general, $H(\omega)$ satisfies

$$H(\omega) + H(\pi - \omega) = 1.$$

- This makes $H(\omega)$ symmetric about the point $\omega = \pi/2$ such that the sum of the values $H(\omega)$ at $\omega = \omega_0 < \pi/2$ and at $\omega = \pi - \omega_0 > \pi/2$ is equal to unity [see Figure (c) in the previous transparency].

Subfilter Approach for Designing Half-Band Filters

- According to the previous discussion, a half-band filter transfer function of order $2M$ with M odd is expressible as

$$H(z) = \frac{1}{2}z^{-M} + G(z^2),$$

where $G(z)$ is a Type II transfer function of odd order M (having allways a zero at $z = -1$).

- Furthermore, the design of $H(z)$ in such a way that $H(\omega)$ approximates on $[0, \omega_p]$ unity with deviation δ and on $[\pi - \omega_p, \ \pi]$ zero with the same deviation δ can be converted into the design of $G(z)$ such that $G(\omega)$ approximates $1/2$ on $[0, \ 2\omega_p]$ with deviation δ.

- In order to construct such a transfer function without general multipliers also for a small value of δ, we generate $G(z)$ as follows:

$$G(z) = \sum_{l=0}^{L} a_l z^{(L-l)K} [F(z)]^{2l+1}, \qquad (A)$$

where

$$F(z) = \sum_{n=0}^{K} f[n]z^{-n}, \quad f[K-n] = f[n].$$

- Here, $F(z)$ is a Type II transfer function of odd order K.
- The order of $G(z)$ is thus $(2L+1)K$ and the delay of each term in the summation of Eq. (A) is $(2L+1)K/2$, as is desired to guarantee the linear-phase performance.
- Efficient implementations of the proposed overall filter for decimation and interpolation purposes are depicted in the following transparency.
- The zero-phase frequency response of the above $G(z)$ is expressible as

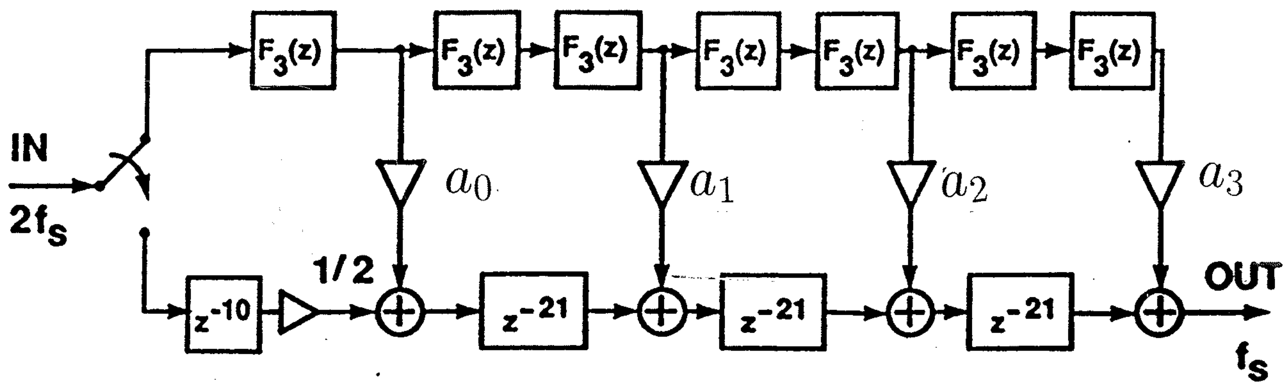
$$G(\omega) = \sum_{l=0}^{L} a_l [F(\omega)]^{2l+1},$$

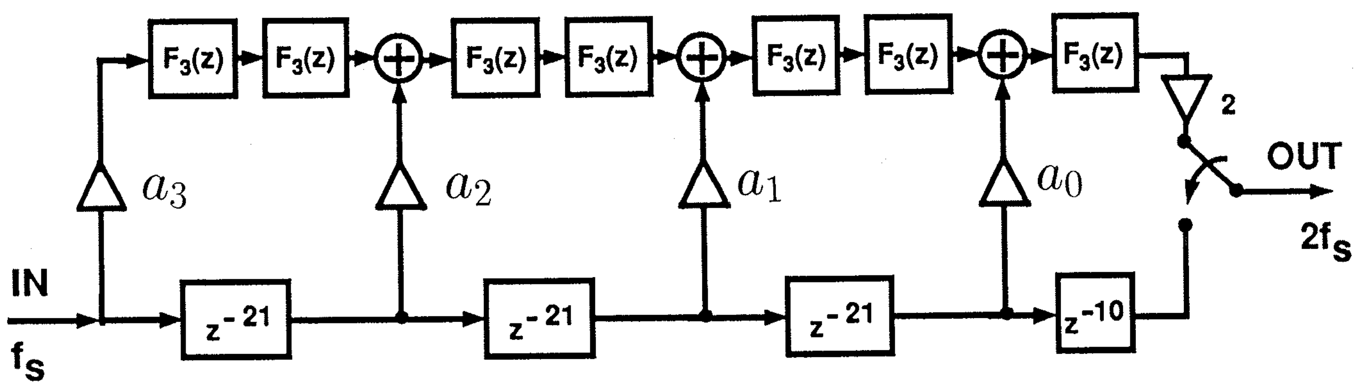
where

$$F(\omega) = 2 \sum_{n=0}^{(K-1)/2} f[(K-1)/2 - n] \cos[(n+1/2)\omega].$$

Implentations of the Proposed Half-Band Filters for Sampling Rate Alteration by a Factor of Two

- The first and second figures show the decimator and interpolator structures for $L = 3$ and $K = 21$.
- These are commutative structures where delay terms have been shared $(F(z) \equiv F_3(z))$.





Simple Design Technique

- Here, we concentrate on the cases with $L = 1$, $L = 2$, and $L = 3$.
- For $L = 3$,

$$G(z) = a_0 F(z) + a_1[F(z)]^3 + a_2[F(z)]^5 + a_3[F(z)]^7$$

and

$$G(\omega) = a_0 F(\omega) + a_1[F(\omega)]^3 + a_2[F(\omega)]^5 + a_3[F(\omega)]^7.$$

- For $L = 2$, $a_3 \equiv 0$ and for $L = 1$, $a_3 = a_2 \equiv 0$.
- Assume that $F(\omega)$ oscillates within $1 - \epsilon_1$ and $1 + \epsilon_2$ with $\epsilon_1 > 0$ and $\epsilon_2 > 0$ on $[0,\ 2\omega_p]$.
- Then, we state the following problem: Given L and δ, find the adjustable parameters a_l as well as ϵ_1 and ϵ_2 such that $G(\omega)$ oscillates within $1/2 \pm \delta$ on $[0,\ 2\omega_p]$.
- If the value of $F(\omega)$ is $1 + \epsilon$, where ϵ is either positive or negative, then the corresponding value of $G(\omega)$ is

$$G(\omega) = a_0[1+\epsilon] + a_1[1+\epsilon]^3 + a_2[1+\epsilon]^5 + a_3[1+\epsilon]^7,$$

where

$$[1+\epsilon]^3 = 1 + 3\epsilon + \epsilon^2,$$

$$[1+\epsilon]^5 = 1 + 5\epsilon + 10\epsilon^2 + 10\epsilon^3 + \epsilon^4,$$

and

$$[1+\epsilon]^7 = 1 + 7\epsilon + 21\epsilon^2 + 35\epsilon^3 + 35\epsilon^4 + 21\epsilon^5 + 7\epsilon^6 + \epsilon^7$$

- For $L = 1$, the selection

$$a_0 = 3/4, \quad a_1 = -1/4$$

gives

$$G(\omega) = 1/2 + \Delta,$$

where

$$\Delta = -(3/4)\epsilon^2 + \epsilon^3,$$

that is, the constant coefficient is equal to 1/2 and the coefficient of ϵ is zero.

- For $L = 2$, the selection

$$a_0 = 15/16, \quad a_1 = -10/16, \quad a_2 = 3/16$$

gives

$$G(\omega) = 1/2 + \Delta,$$

where

$$\Delta = -(5/4)\epsilon^3 + (15/16)\epsilon^4 + (3/16)\epsilon^5,$$

that is, the constant coefficient is equal to 1/2 and the coefficients of ϵ and ϵ^2 are zero.

- For $L = 3$, the selection

$$a_0 = 35/32, \;\; a_1 = -35/32, \;\; a_2 = 21/32, \;\; a_3 = -5/32$$

gives

$$G(\omega) = 1/2 + \Delta,$$

where

$$\Delta = -(35/16)\epsilon^4 - (21/8)\epsilon^5 - (35/32)\epsilon^6 - (5/32)\epsilon^7,$$

that is, the constant coefficient is equal to 1/2 and the coefficients of ϵ, ϵ^2, and ϵ^3 are zero.

- In all the above cases, the variation of $G(\omega)$ around 1/2, denoted by Δ, is significantly smaller than the variation of $F(\omega)$ around unity, ϵ.

- In the following, we consider two cases. In the first case, referred to as Case A, it is required that $-0.001 \leq \Delta \leq 0.001$. In the second case, referred to as Case B, it is required that $-0.000001 \leq \Delta \leq 0.000001$.

- In Cases A and B, the stopband attenuations of the overall half-band filters are 60 dB and 120 dB, respectively.

$L = 1,\ a_0 = 2^{-1} + 2^{-2},\ a_1 = -2^{-2}$

- In this case, the deviation of $G(\omega)$ from 1/2, denoted by Δ, is related to the deviation of $F(\omega)$ from unity, denoted by ϵ, through the equation

$$\Delta = -(3/4)\epsilon^2 + \epsilon^3.$$

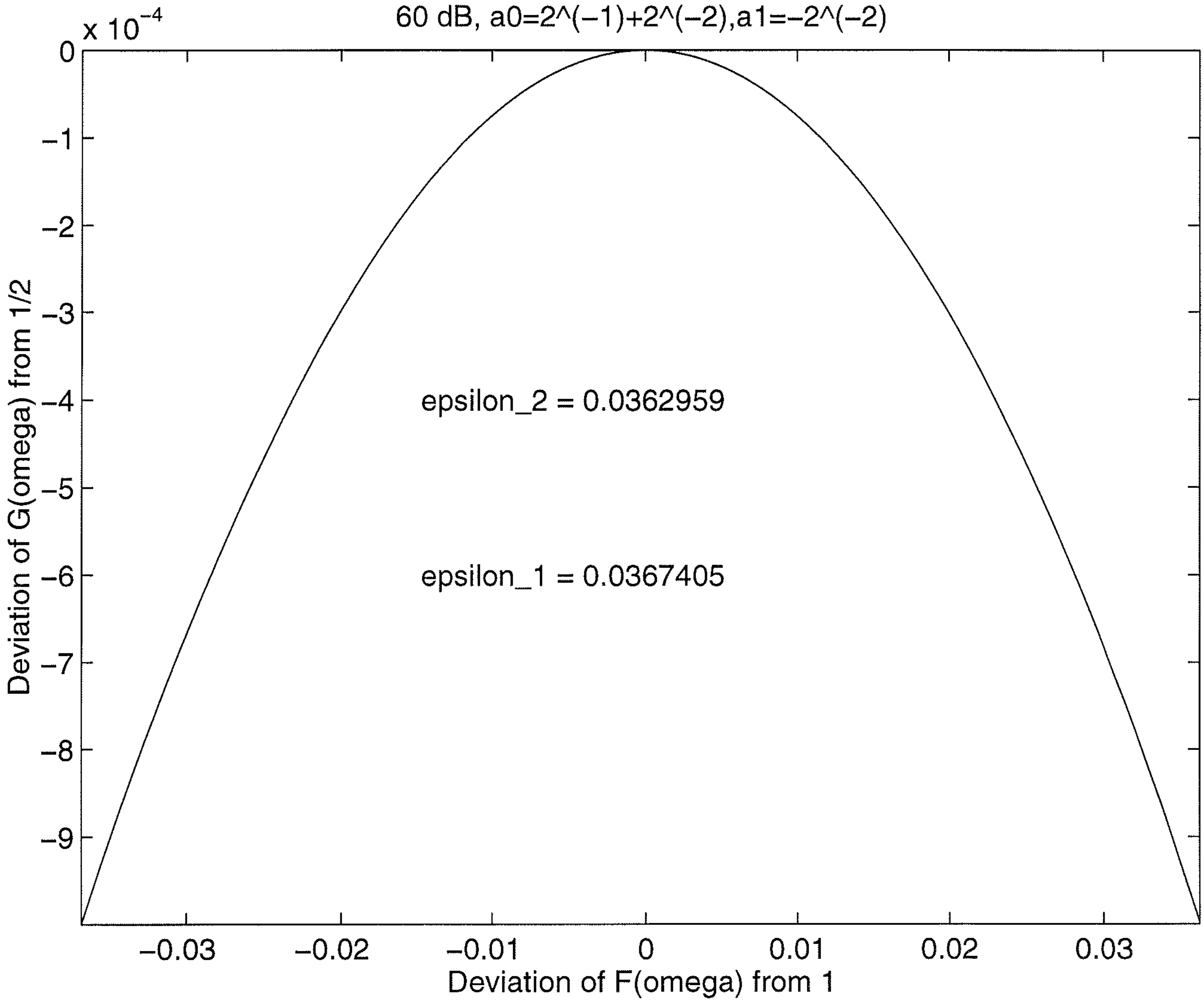
- The following two transparencies give plots of the above equation for both Case A ($|\Delta| \leq 0.001$) and Case B ($|\Delta| \leq 0.000001$).
- It is seen that in Case A $F(\omega)$ is allowed to vary within the limits $1 - \epsilon_1$ and $1 + \epsilon_2$, where

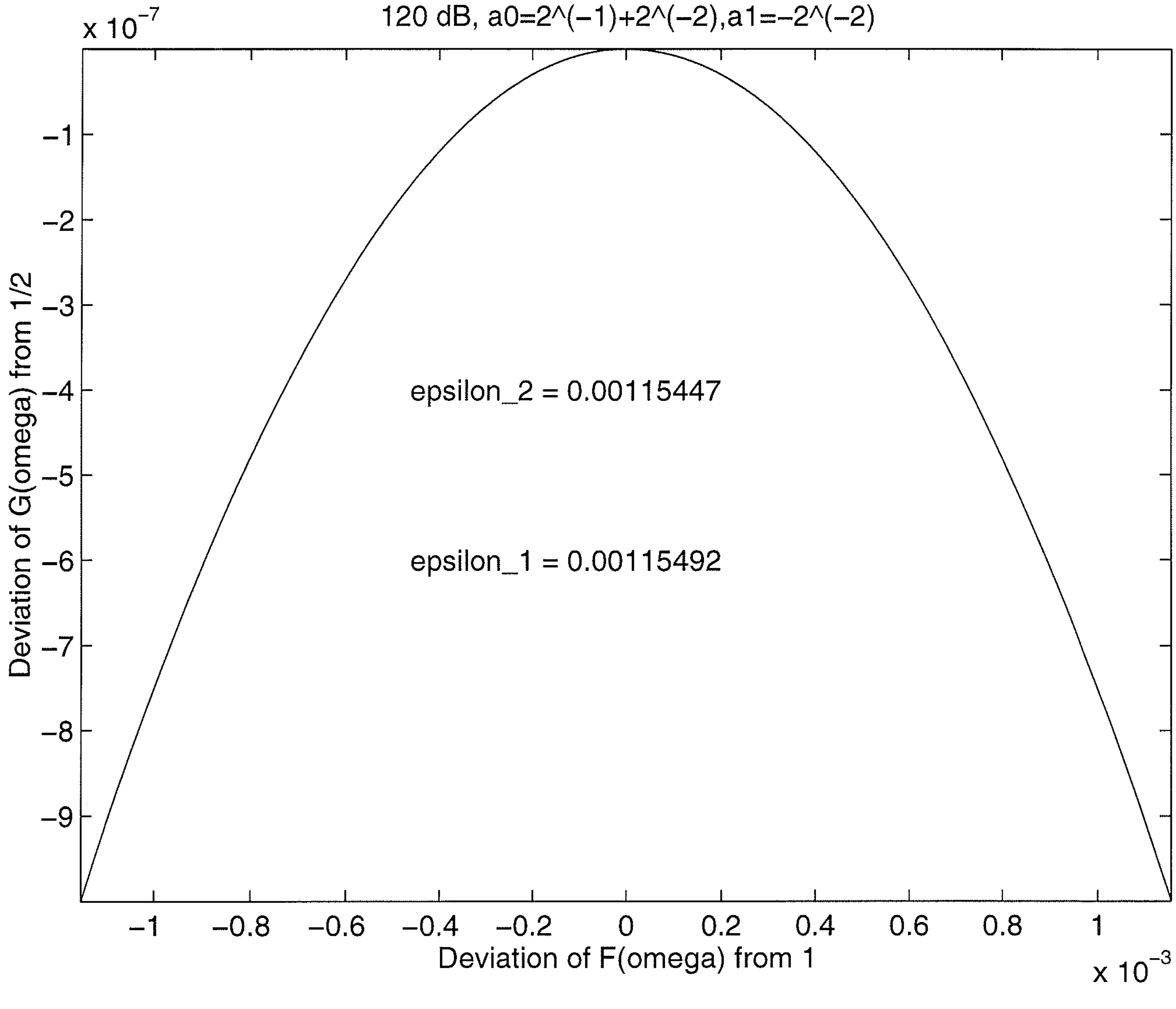
$$\epsilon_1 = 0.0367405, \quad \epsilon_2 = 0.0362959,$$

to satisfy $-0.001 \leq \Delta \leq 0.001$.
- In Case B, $F(\omega)$ is allowed to vary within the limits $1 - \epsilon_1$ and $1 + \epsilon_2$, where

$$\epsilon_1 = 0.00115492, \quad \epsilon_2 = 0.00115447,$$

to satisfy $-0.000001 \leq \Delta \leq 0.000001$.
- The disadvantage of the above selections of a_0 and a_1 is that the maximum value of Δ is zero.


60 dB, a0=2^(−1)+2^(−2),a1=−2^(−2)
x 10^-4
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
Deviation of G(omega) from 1/2
epsilon_2 = 0.0362959
epsilon_1 = 0.0367405
−0.03
−0.02
−0.01
0
0.01
0.02
0.03
Deviation of F(omega) from 1


120 dB, a0=2^(−1)+2^(−2),a1=−2^(−2)
x 10^−7
−1
−2
−3
−4
−5
−6
−7
−8
−9
Deviation of G(omega) from 1/2
epsilon_2 = 0.00115447
epsilon_1 = 0.00115492
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Deviation of F(omega) from 1
x 10^−3

- Better results, that is, both ϵ_1 and ϵ_2 become larger, is obtained by changing a_1. This is considered next.

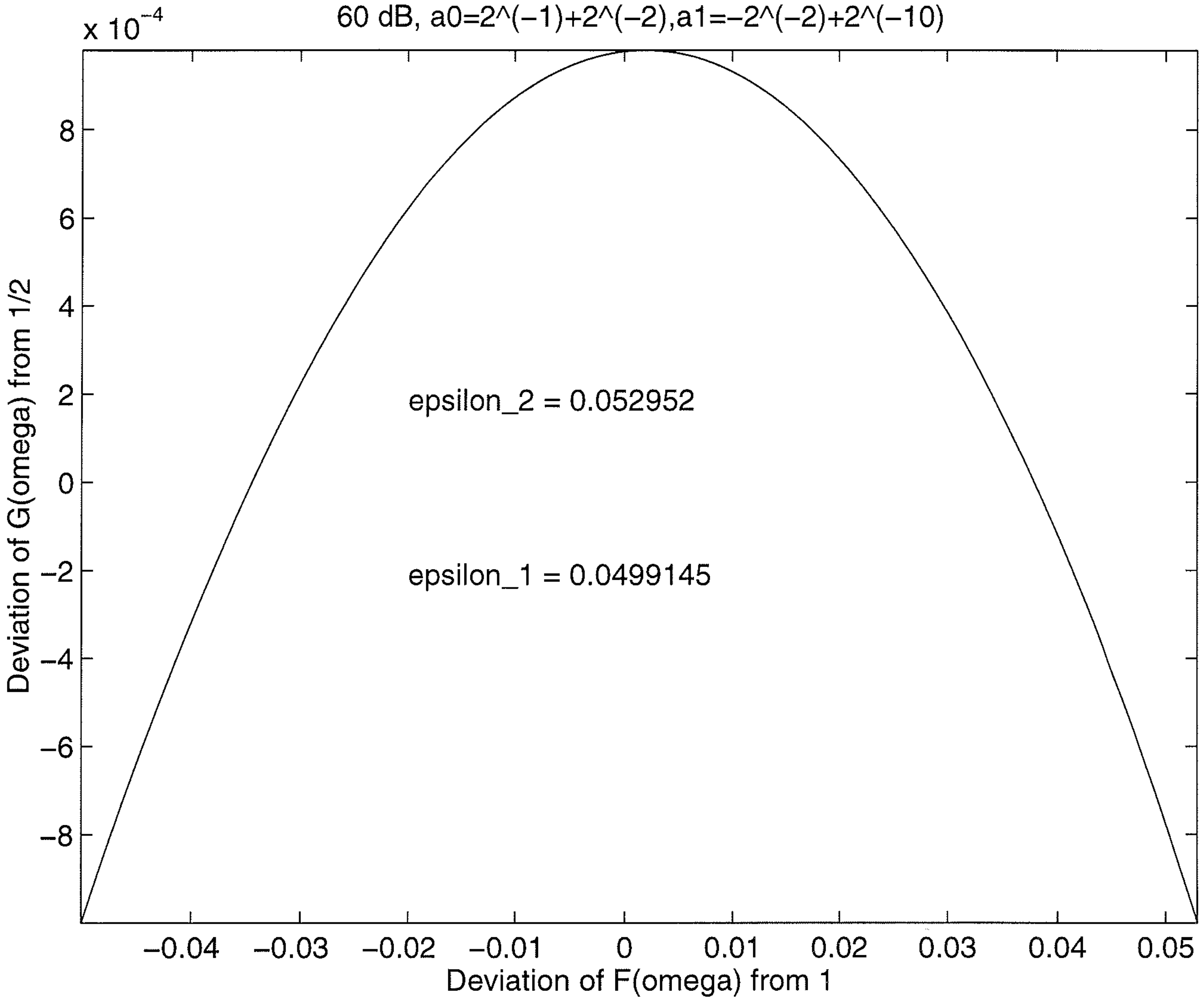
Case A: $L = 1,\ a_0 = 2^{-1} + 2^{-2},\ a_1 = -2^{-2} + 2^{-10}$

- In this case,

$$\Delta = -(3/4)\epsilon^2 + \epsilon^3 + 2^{-10}(1+\epsilon)^3.$$

- As seen from the following transparency, in this case

$$\epsilon_1 = 0.0499145, \quad \epsilon_2 = 0.0529520.$$


60 dB, a0=2^(-1)+2^(-2),a1=-2^(-2)+2^(-10)
x 10^-4
8
6
4
2
0
-2
-4
-6
-8
Deviation of G(omega) from 1/2
epsilon_2 = 0.052952
epsilon_1 = 0.0499145
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
Deviation of F(omega) from 1

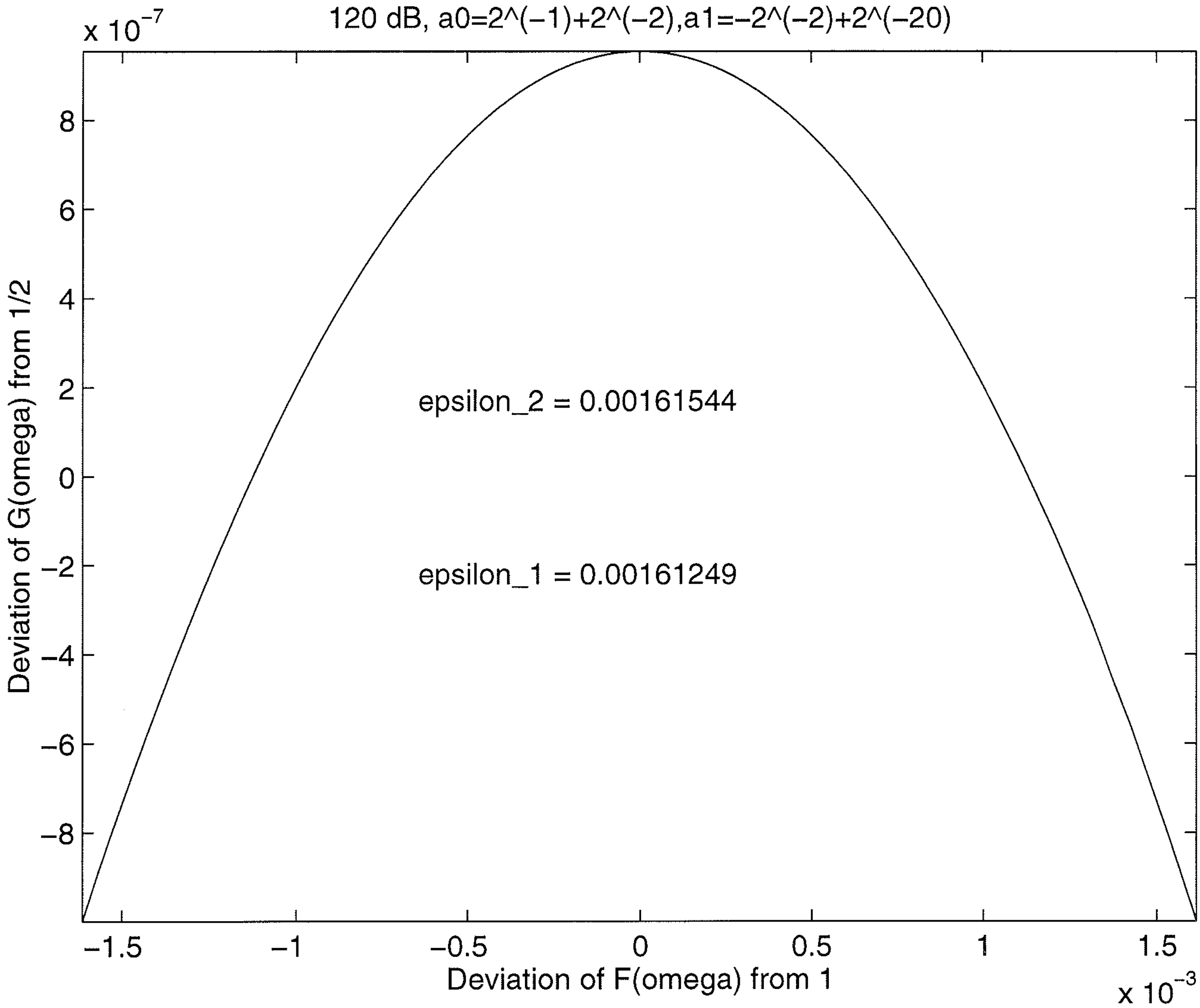
Case B: $L = 1,\ a_0 = 2^{-1} + 2^{-2},\ a_1 = -2^{-2} + 2^{-20}$

- In this case,

$$\Delta = -(3/4)\epsilon^2 + \epsilon^3 + 2^{-20}(1+\epsilon)^3.$$

- As seen from the following transparency, in this case

$$\epsilon_1 = 0.00161249, \quad \epsilon_2 = 0.00161544.$$



120 dB, a0=2^(-1)+2^(-2),a1=-2^(-2)+2^(-20)
x 10^-7
8
6
4
2
0
-2
-4
-6
-8
Deviation of G(omega) from 1/2
epsilon_2 = 0.00161544
epsilon_1 = 0.00161249
-1.5
-1
-0.5
0
0.5
1
1.5
Deviation of F(omega) from 1
x 10^-3

$L = 2,\ a_0 = 2^0 - 2^{-4},\ a_1 = -2^{-1} - 2^{-3},\ a_2 = 2^{-3} + 2^{-4}$

- In this case,

$$\Delta = -(5/4)\epsilon^3 + (15/16)\epsilon^4 + (3/16)\epsilon^5.$$

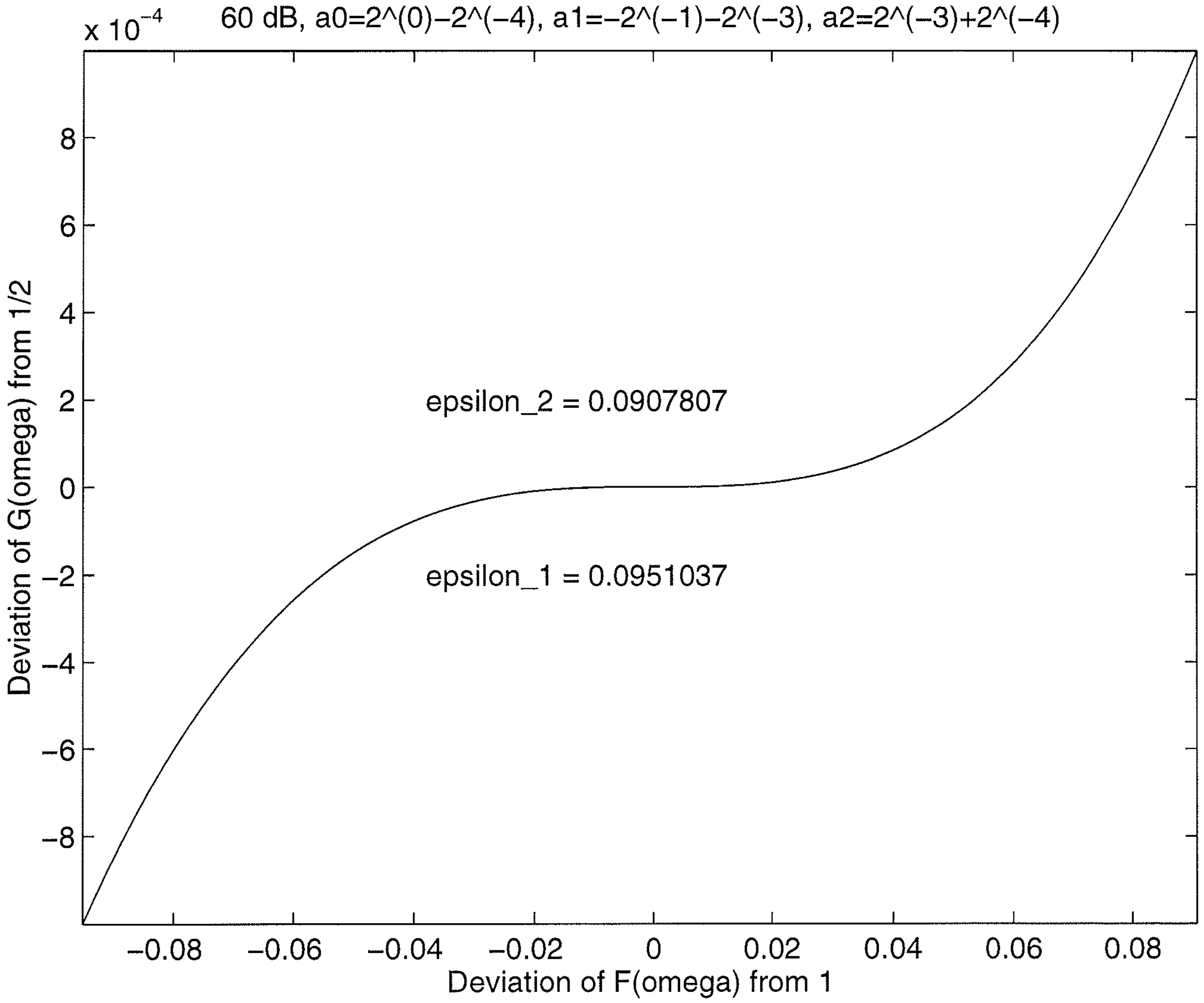
- As seen from the following two transparencies, in Case A,

$$\epsilon_1 = 0.0951037, \quad \epsilon_2 = 0.0907807,$$

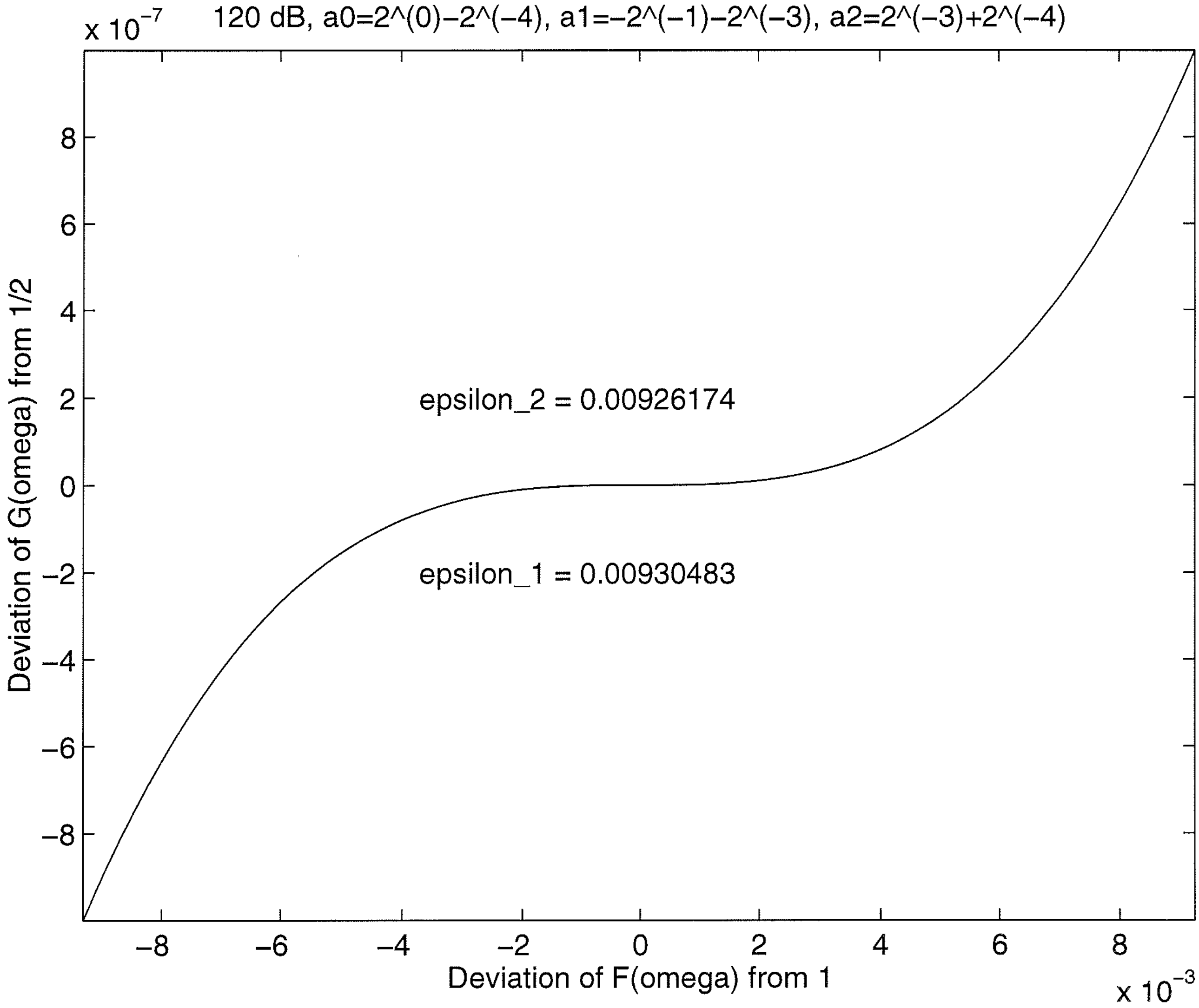
whereas in Case B,

$$\epsilon_1 = 0.00930483, \quad \epsilon_2 = 0.00926174.$$

- In these cases, Δ takes both positive and negative values.



60 dB, a0=2^(0)−2^(−4), a1=−2^(−1)−2^(−3), a2=2^(−3)+2^(−4)
x 10^−4
8
6
4
2
0
−2
−4
−6
−8
epsilon_2 = 0.0907807
epsilon_1 = 0.0951037
−0.08
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
0.08
Deviation of F(omega) from 1
Deviation of G(omega) from 1/2



x 10^-7
120 dB, a0=2^(0)−2^(−4), a1=−2^(−1)−2^(−3), a2=2^(−3)+2^(−4)
8
6
4
2
0
−2
−4
−6
−8
Deviation of G(omega) from 1/2
epsilon_2 = 0.00926174
epsilon_1 = 0.00930483
−8
−6
−4
−2
0
2
4
6
8
Deviation of F(omega) from 1
x 10^-3

$L = 3,\ a_0 = 2^0 + 2^{-4} + 2^{-5},\ a_1 = -2^0 - 2^{-4} - 2^{-5},$
$a_2 = 2^{-1} + 2^{-3} + 2^{-5},\ a_3 = -2^{-3} - 2^{-5}$

- In this case,

$$\Delta = -(35/16)\epsilon^4 - (21/8)\epsilon^5 - (35/32)\epsilon^6 - (5/32)\epsilon^7.$$

- As seen from the following two transparencies, in Case A,

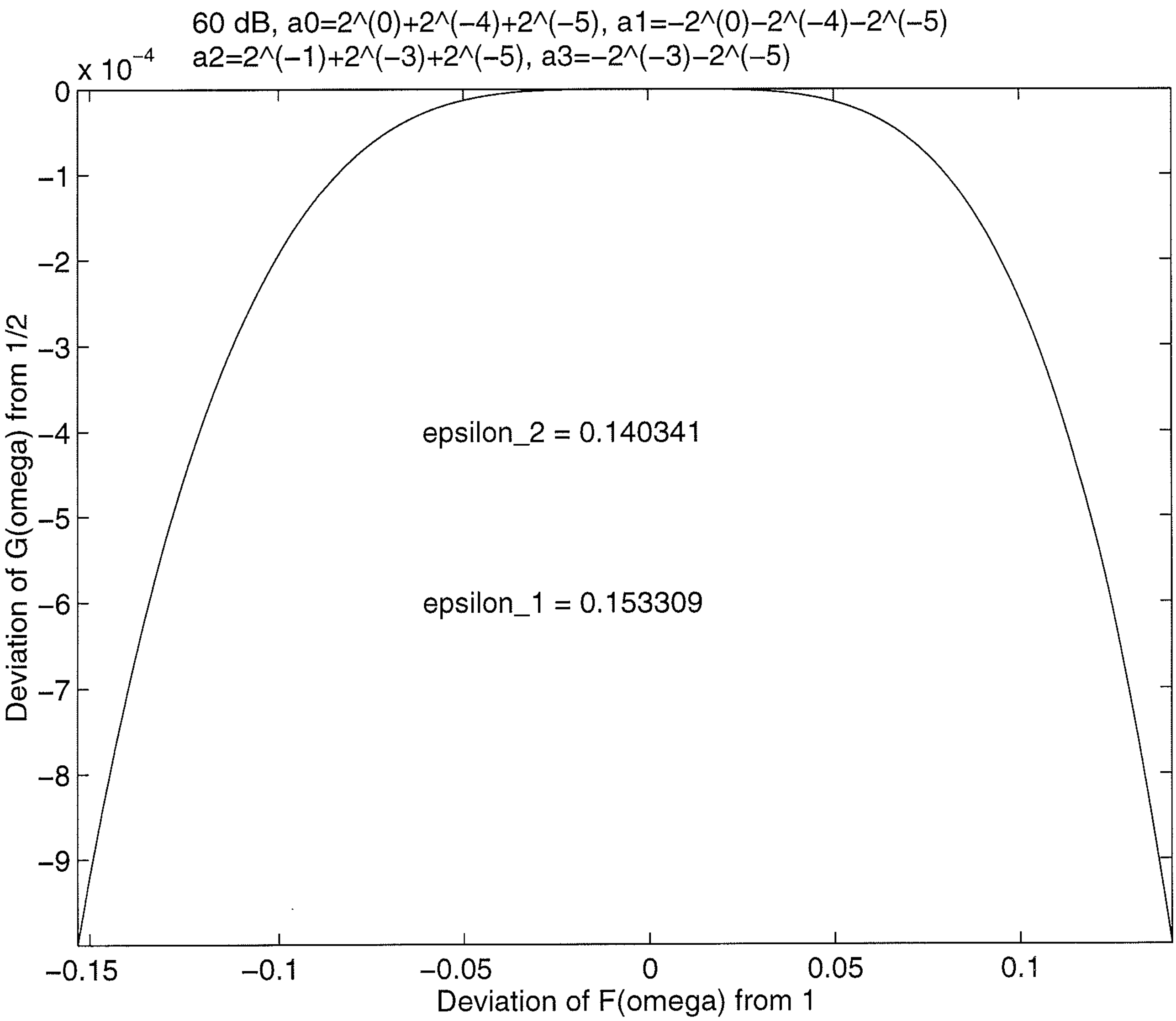
$$\epsilon_1 = 0.153309, \quad \epsilon_2 = 0.140341,$$

whereas in Case B,

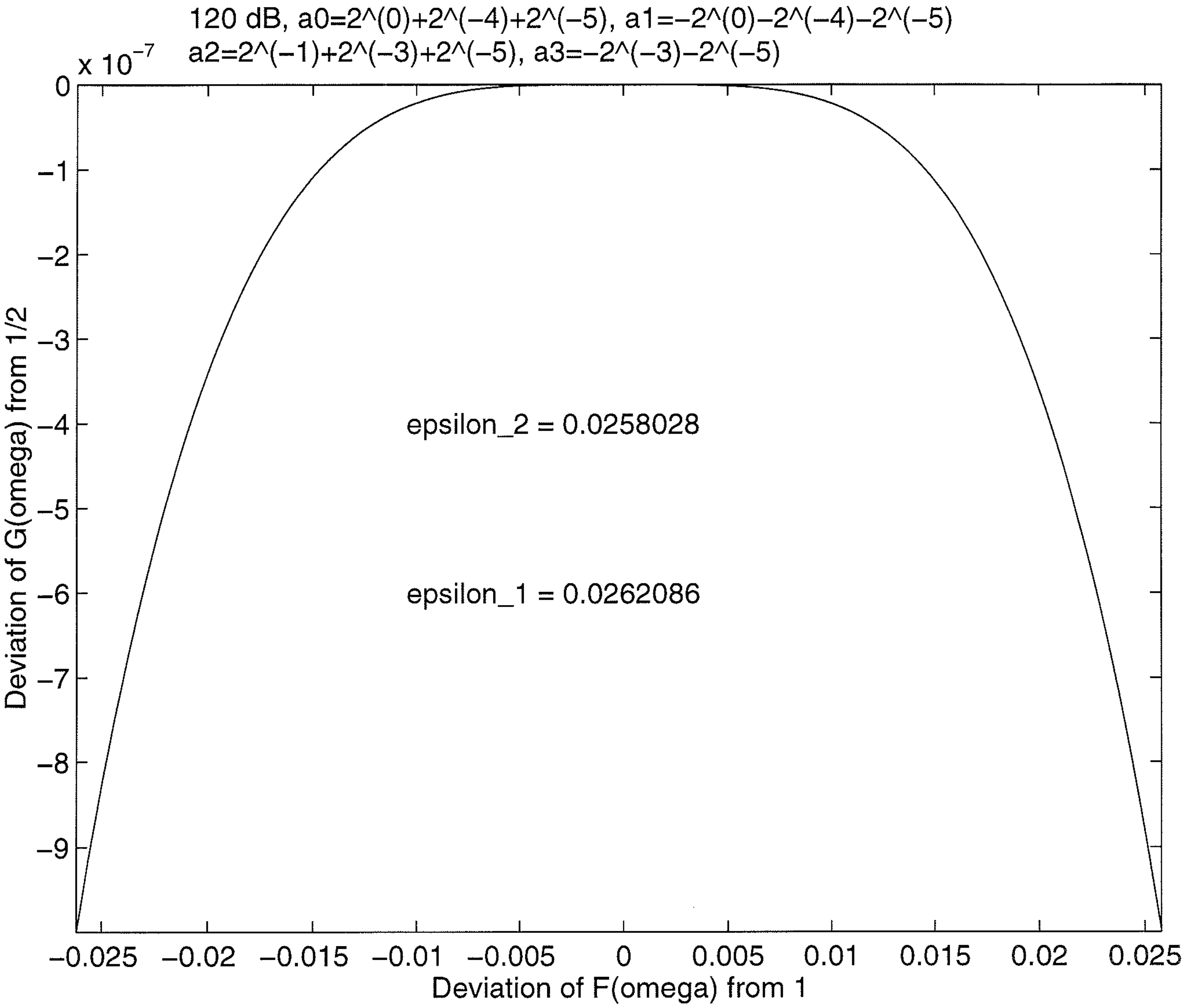
$$\epsilon_1 = 0.0262086, \quad \epsilon_2 = 0.0258028$$

- The disadvantage of the above selections of a_0, a_1, a_2, and a_3 is that the maximum value of Δ is zero.

- Better results, that is, both ϵ_1 and ϵ_2 become larger, is obtained by changing a_3. This is considered next.



60 dB, a0=2^(0)+2^(-4)+2^(-5), a1=-2^(0)-2^(-4)-2^(-5)
a2=2^(-1)+2^(-3)+2^(-5), a3=-2^(-3)-2^(-5)
x 10^-4
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
Deviation of G(omega) from 1/2
epsilon_2 = 0.140341
epsilon_1 = 0.153309
-0.15
-0.1
-0.05
0
0.05
0.1
Deviation of F(omega) from 1



120 dB, a0=2^(0)+2^(-4)+2^(-5), a1=-2^(0)-2^(-4)-2^(-5)
a2=2^(-1)+2^(-3)+2^(-5), a3=-2^(-3)-2^(-5)
x 10^-7
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
Deviation of G(omega) from 1/2
epsilon_2 = 0.0258028
epsilon_1 = 0.0262086
-0.025
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02
0.025
Deviation of F(omega) from 1

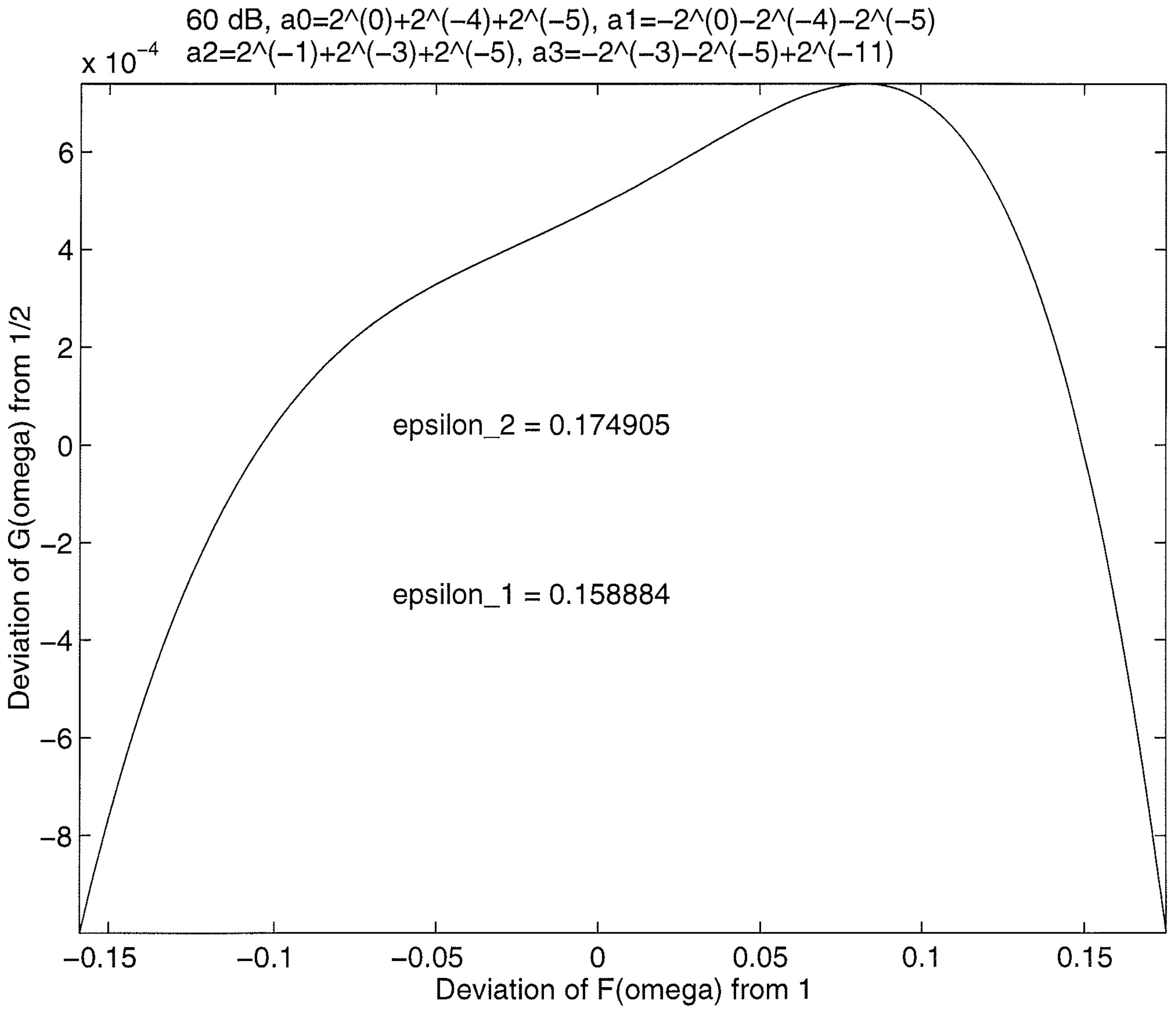
Case A: $L = 3$, $a_0 = 2^0 + 2^{-4} + 2^{-5}$, $a_1 = -2^0 - 2^{-4} - 2^{-5}$, $a_2 = 2^{-1} + 2^{-3} + 2^{-5}$, $a_3 = -2^{-3} - 2^{-5} + 2^{-11}$

- In this case,

$$\Delta = -(35/16)\epsilon^4 - (21/8)\epsilon^5 - (35/32)\epsilon^6 - (5/32)\epsilon^7 + 2^{-11}(1+\epsilon)^7.$$

- As seen from the following transparency, in this case

$$\epsilon_1 = 0.158884, \quad \epsilon_2 = 0.174905.$$


60 dB, a0=2^(0)+2^(-4)+2^(-5), a1=-2^(0)-2^(-4)-2^(-5)
a2=2^(-1)+2^(-3)+2^(-5), a3=-2^(-3)-2^(-5)+2^(-11)
x 10^-4
6
4
2
0
-2
-4
-6
-8
Deviation of G(omega) from 1/2
epsilon_2 = 0.174905
epsilon_1 = 0.158884
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
Deviation of F(omega) from 1

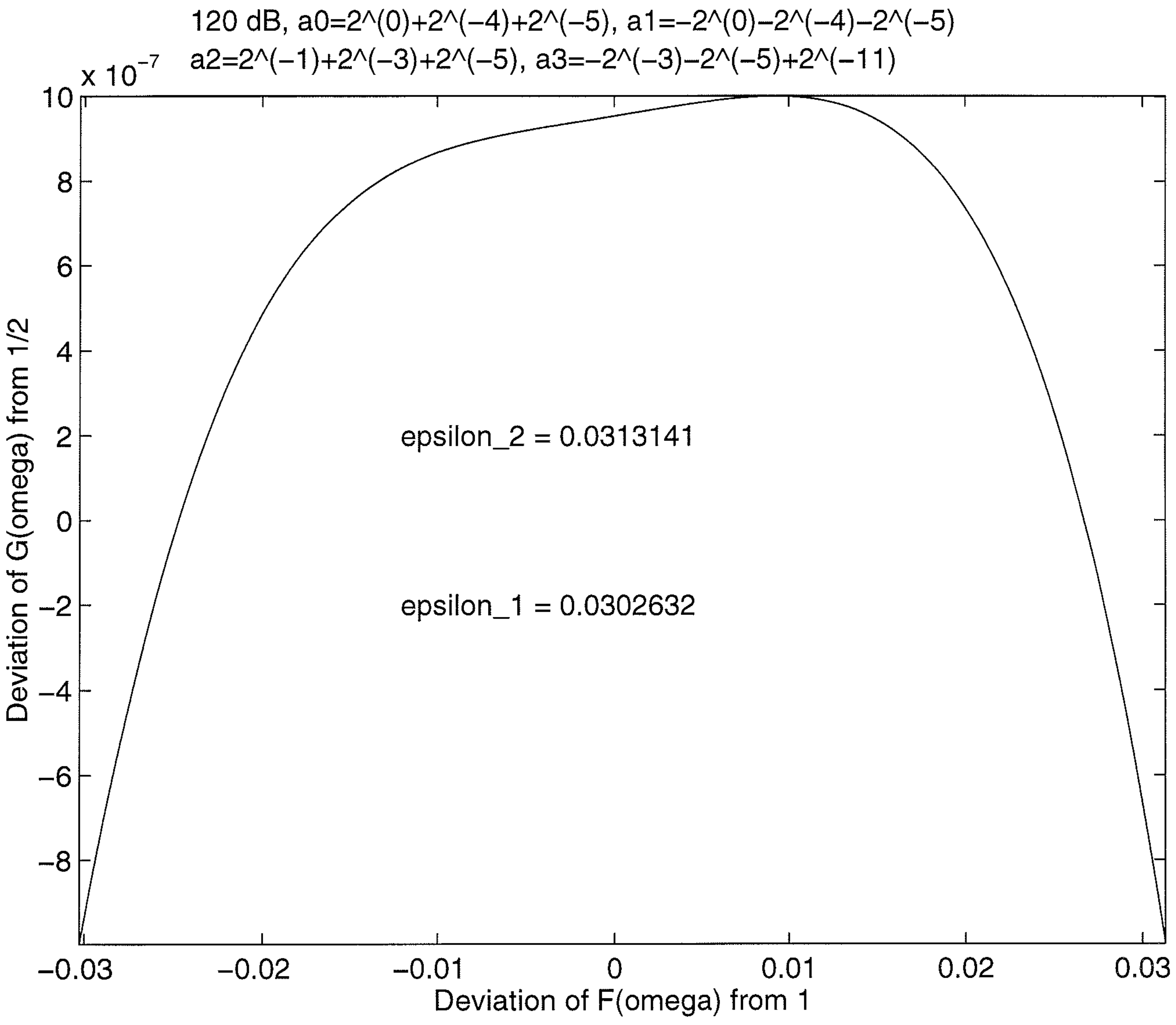
Case B: $L = 3,\ a_0 = 2^0 + 2^{-4} + 2^{-5},\ a_1 = -2^0 - 2^{-4} - 2^{-5},\ a_2 = 2^{-1} + 2^{-3} + 2^{-5},\ a_3 = -2^{-3} - 2^{-5} + 2^{-20}$

- In this case,

$$\begin{aligned}\Delta = & -(35/16)\epsilon^4 - (21/8)\epsilon^5 - (35/32)\epsilon^6 \\ & -(5/32)\epsilon^7 + 2^{-20}(1+\epsilon)^7.\end{aligned}$$

- As seen from the following transparency, in this case

$$\epsilon_1 = 0.0302632, \quad \epsilon_2 = 0.0313141.$$



120 dB, a0=2^(0)+2^(−4)+2^(−5), a1=−2^(0)−2^(−4)−2^(−5)
a2=2^(−1)+2^(−3)+2^(−5), a3=−2^(−3)−2^(−5)+2^(−11)
x 10^−7
10
8
6
4
2
0
−2
−4
−6
−8
Deviation of G(omega) from 1/2
epsilon_2 = 0.0313141
epsilon_1 = 0.0302632
−0.03
−0.02
−0.01
0
0.01
0.02
0.03
Deviation of F(omega) from 1

Example

- It is desired to design a half-band decimator in such a way that
  1) the sampling rate reduction ratio is 2 and the output sampling rate is 44.1 kHz.
  2) Components alising into the band from 0 Hz to 20 kHz are attenuated at least 120 dB.
- In this case the problem is to design $G(z)$ such that the deviation of its zero-phase frequency response $G(\omega)$ from 1/2 is at most 0.000001 in the passband.
- The passband edge is at

$$2\omega_p = [20/(44.1/2)]\pi = 0.90703\pi.$$

- We select $L = 3$, $a_0 = 2^0 + 2^{-4} + 2^{-5}$, $a_1 = -2^0 - 2^{-4} - 2^{-5}$, $a_2 = 2^{-1} + 2^{-3} + 2^{-5}$, and $a_3 = -2^{-3} - 2^{-5} + 2^{-20}$.
- In this case, the problem is to design an odd order $F(z)$ such that its zero-phase frequency response $F(\omega)$ stays within the limits $1 - \epsilon_1$ and

$1+\epsilon_2$ on $[0,\ 2\omega_p]$ with

$$\epsilon_1 = 0.0302632, \quad \epsilon_2 = 0.0313141.$$

- The design can be accomplished with the aid of the Remez algorithm by using a single band $[0,\ 2\omega_p]$. The desired function is $(2-\epsilon_1+\epsilon_2) = 1.00052545$, whereas the allowable deviation is $1+\epsilon_2 - 1.00052545 = 0.03078865$.

- The given criteria are met by $F(z)$ of order 21 and having the impulse-response coefficient values

$$f[0] = f[21] = 2^{-6},\ f[1] = f[20] = -2^{-7} - 2^{-8},$$

$$f[2] = f[19] = 2^{-6},\ f[3] = f[18] = 2^{-6} - 2^{-7},$$

$$f[4] = f[17] = 2^{-5},\ f[5] = f[16] = -2^{-5} - 2^{-7} - 2^{-8},$$

$$f[6] = f[15] = 2^{-4} - 2^{-8},\ f[7] = f[14] = -2^{-4} - 2^{-6} - 2^{-8},$$

$$f[8] = f[13] = 2^{-3} - 2^{-8},\ f[9] = f[12] = -2^{-2} + 2^{-5} + 2^{-7},$$

and

$$f[10] = f[11] = 2^{-1} + 2^{-3} + 2^{-7}.$$

- These values have been obtained by first rounding the coefficients of $F(z)$ to 8 fractional bits.

This rounding gives $f[10] = f[11] = 163 \cdot 2^{-8}$ and $f[9] = f[12] = -53 \cdot 2^{-8}$, which are not expressible as three powers of two. Therefore, they are rounded to the nearest three powers-ot-two respresentations, giving $f[10] = f[11] = 162 \cdot 2^{-8}$ and $f[9] = f[12] = -54 \cdot 2^{-8}$.

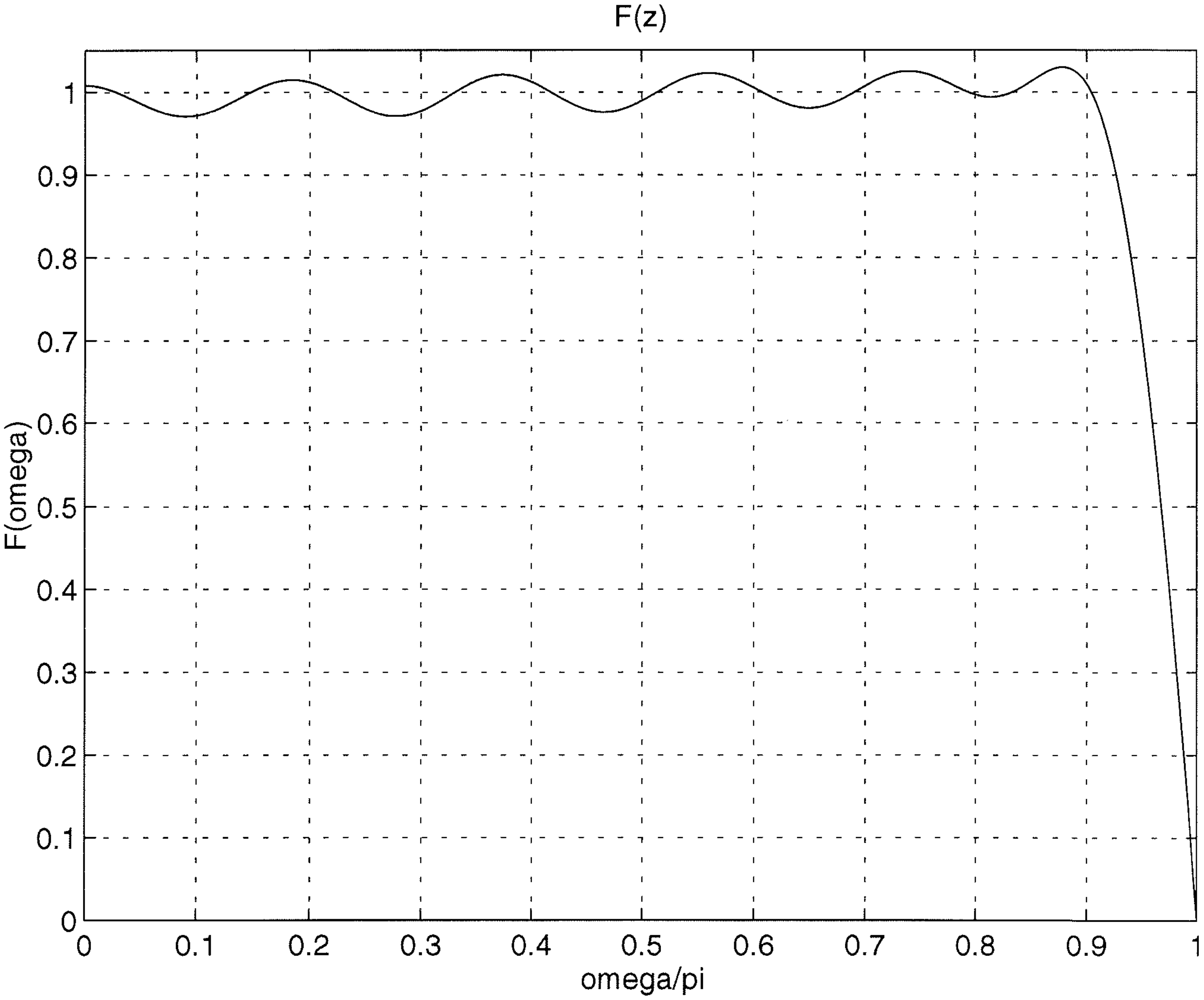
- In the following, there are six transparencies illustrating the chracteristics of $F(z)$, $G(z)$, and the resulting overall half-band FIR filter $H(z)$ with passband and stopband edges at $\omega_p = 0.453515\pi$ and $\omega_s = \pi - \omega_p = 0.546485\pi$ and a 120-dB attenuation in the stopband.

- In addition, there are two Matlab-files, halquan.m and half.m. The first file finds the above $F(z)$, whereas the second file plots the responses for $F(z)$, $G(z)$, and $H(z)$.

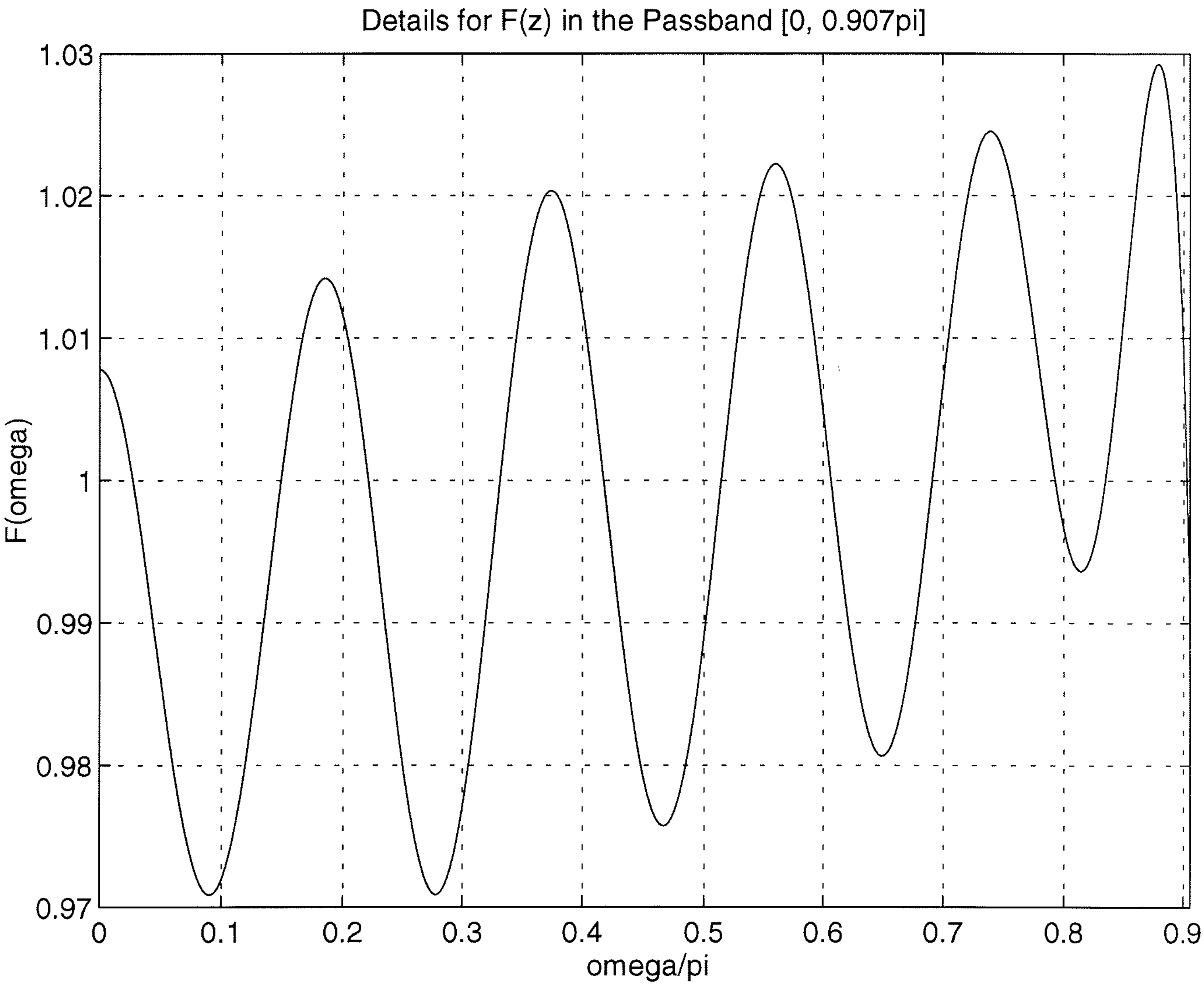
- In the very end of these lecture notes there are the following article as well as the corresponding conference talk:

- T. Saramäki, T. Karema, T. Ritoniemi, and H.

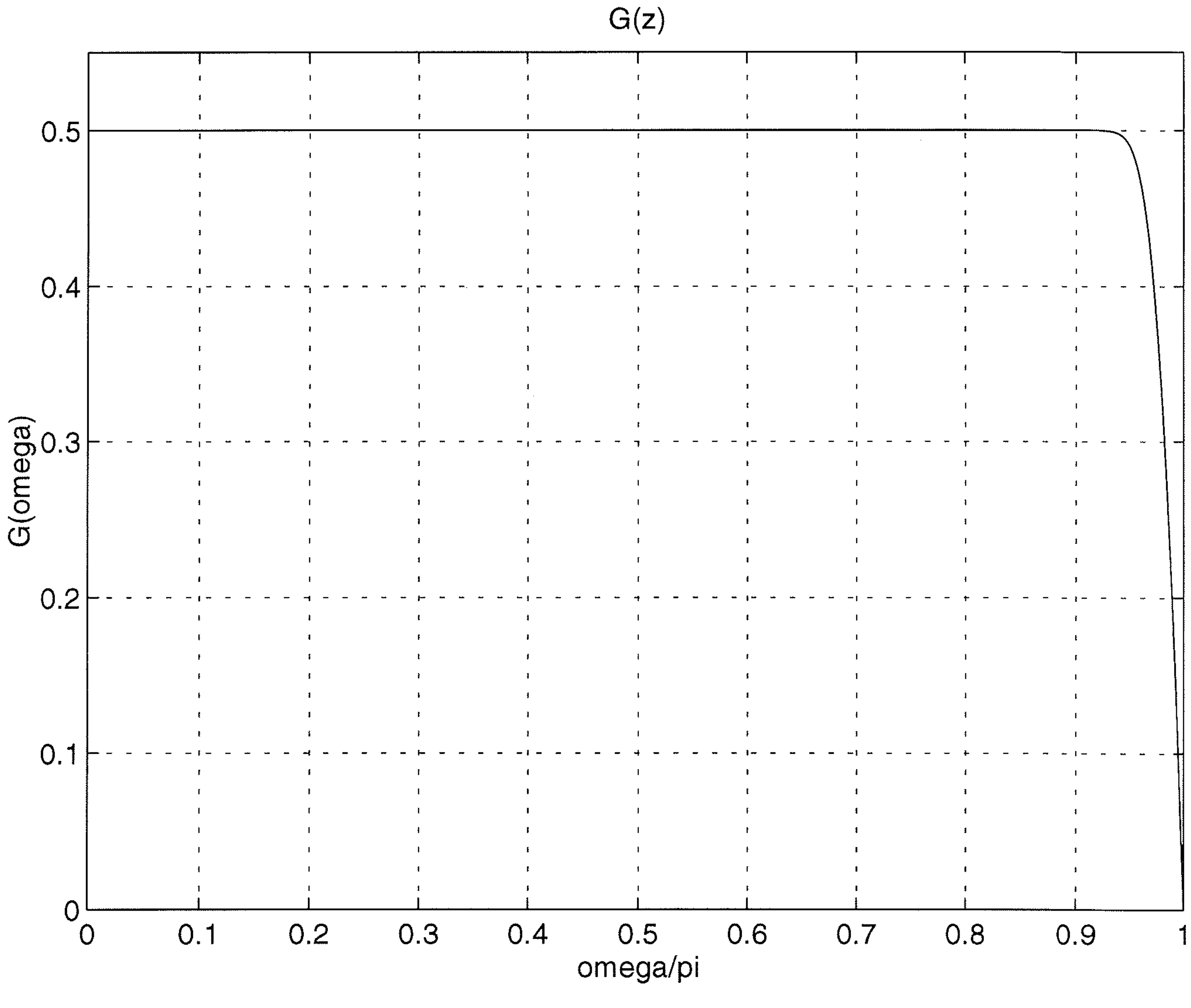
Tenhunen, "Multiplier-free decimator algorithms for superresolution oversampled converters," in *Proc.* 1990 *IEEE International Symposium on Circuits and Systems* (New Orleans, Louisiana), pp. 3275–3278, May 1990.

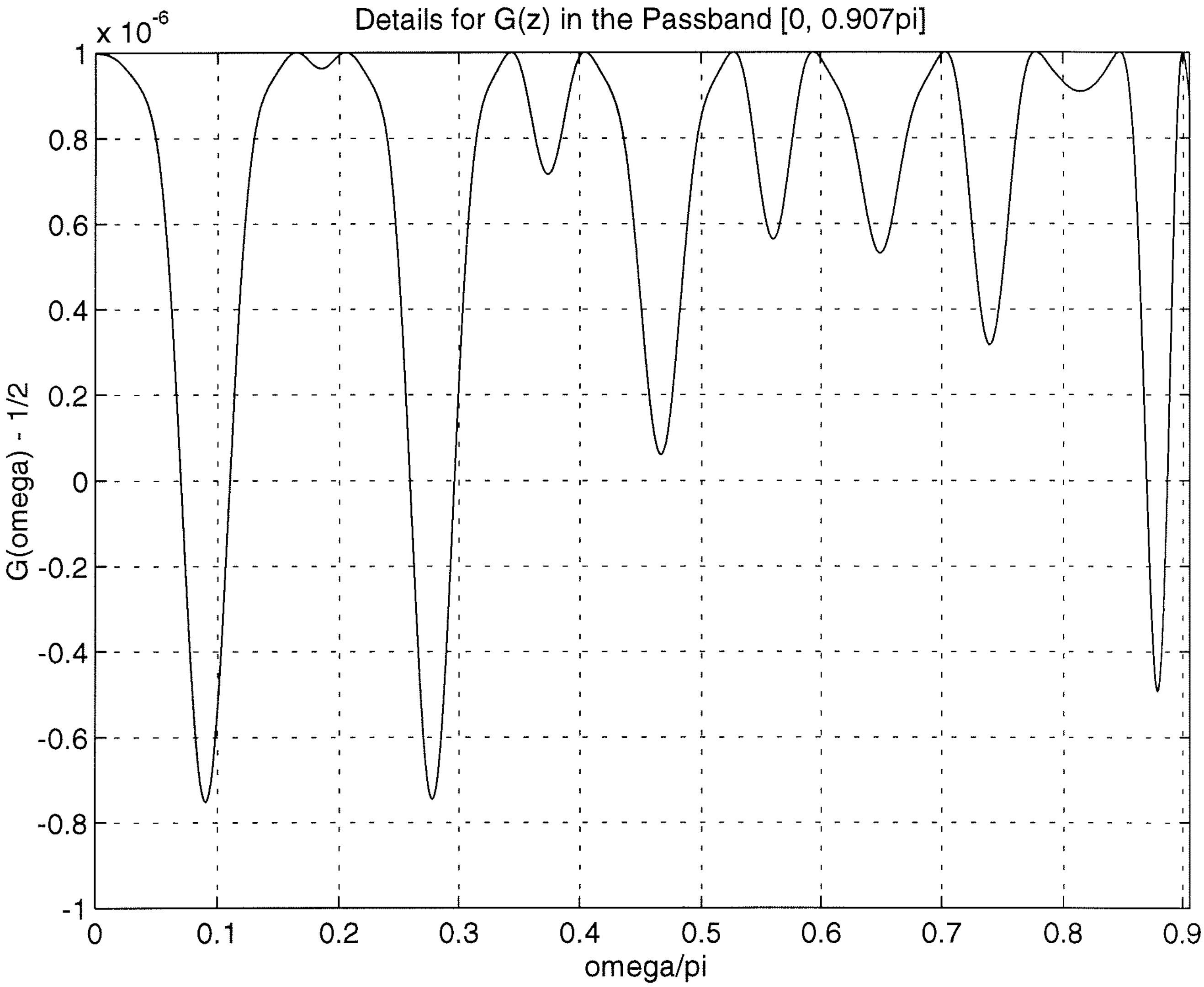


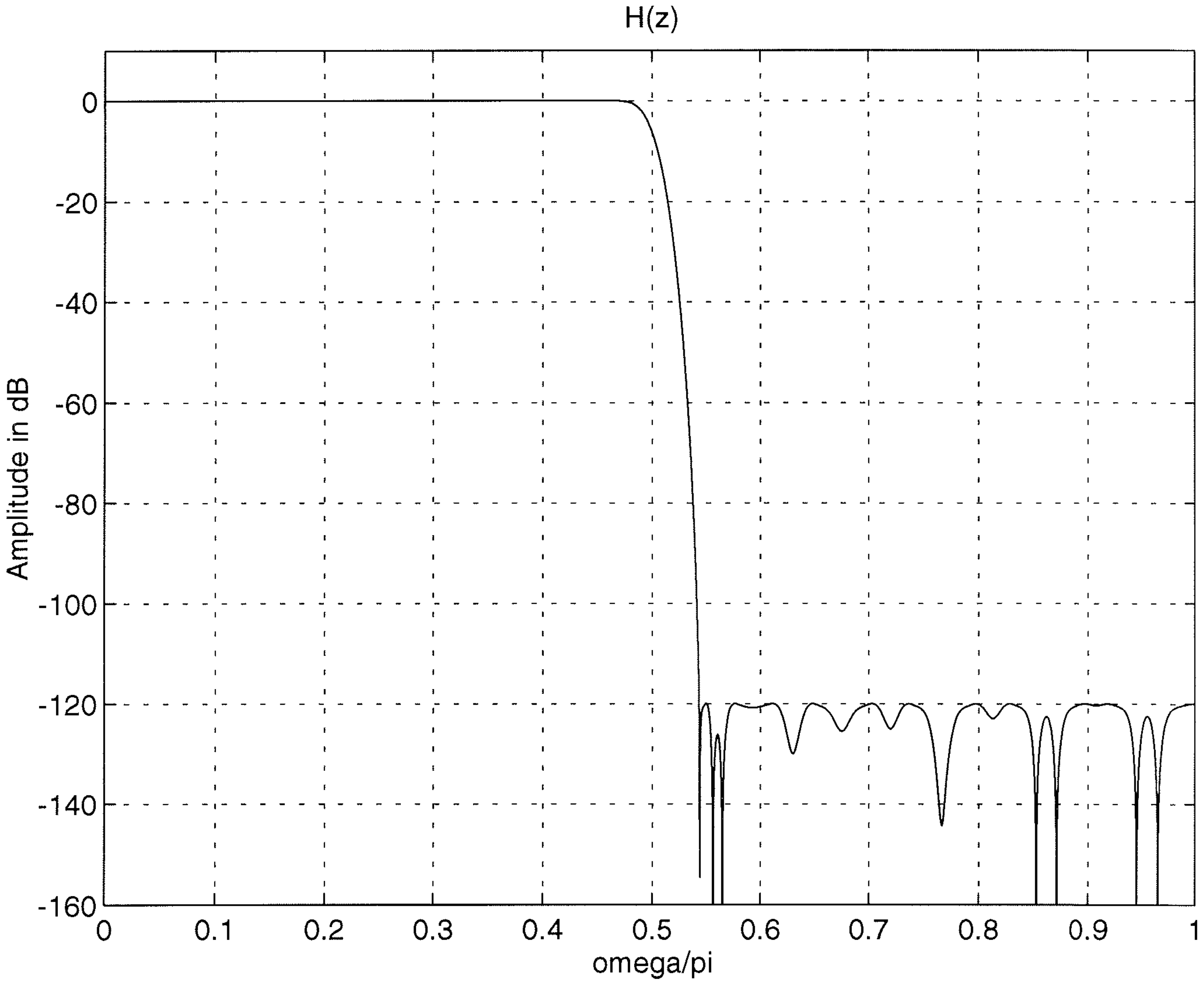
F(z)
F(omega)
omega/pi
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

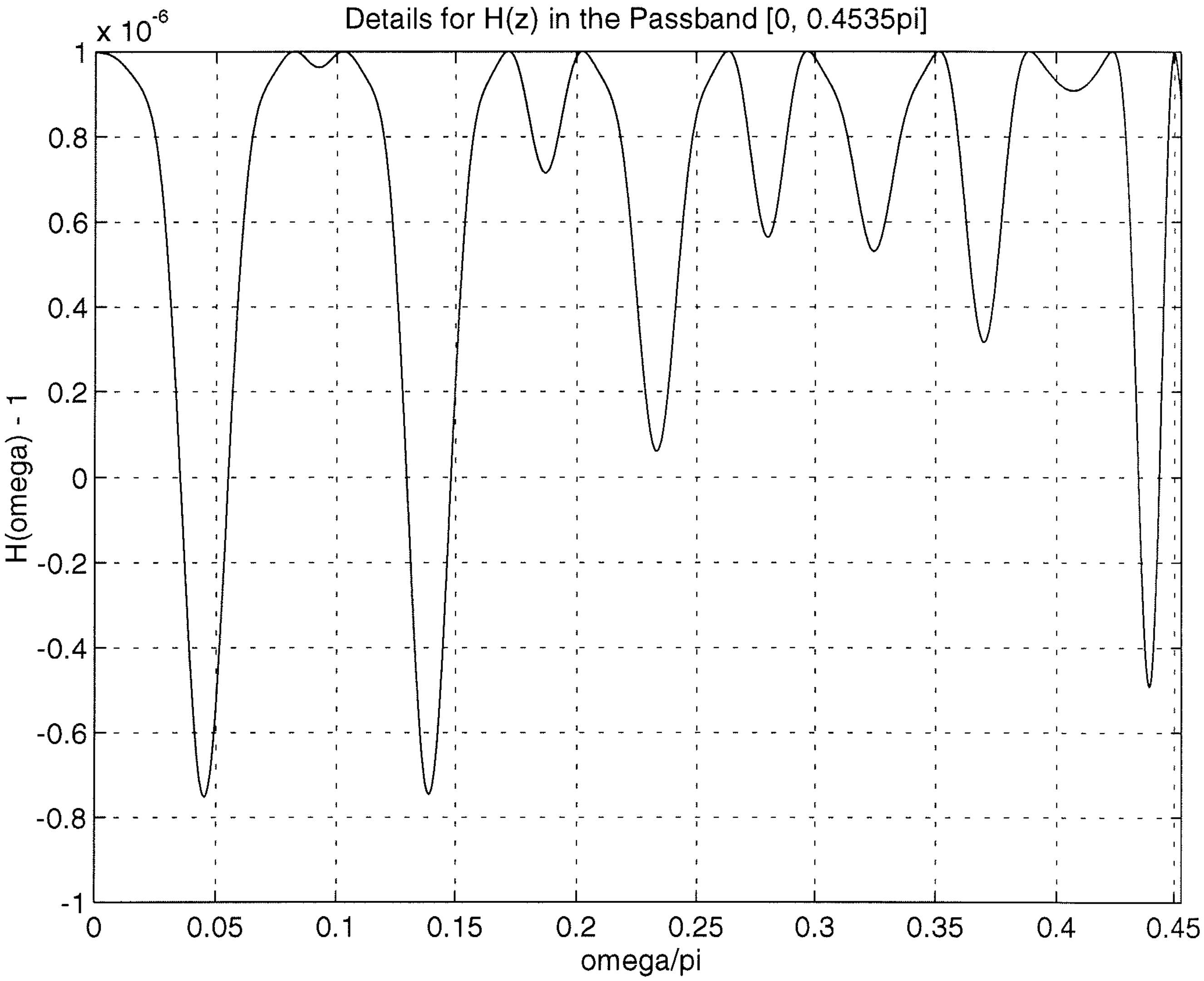


Details for F(z) in the Passband [0, 0.907pi]
1.03
1.02
1.01
1
0.99
0.98
0.97
F(omega)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
omega/pi


G(z)
0.5
0.4
0.3
0.2
0.1
0
G(omega)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
omega/pi


Details for G(z) in the Passband [0, 0.907pi]
x 10^-6
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
G(omega) - 1/2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
omega/pi


H(z)
Amplitude in dB
0
-20
-40
-60
-80
-100
-120
-140
-160
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
omega/pi



Details for H(z) in the Passband [0, 0.4535pi]
x 10^-6
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
H(omega) - 1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
omega/pi

```
% Matlab-file halquan.m for designing
% the example F(z)
% can be found in SUN's: ~ts/matlab/sldsp
close all;clear all
N=21;e1=0.0302632;e2=0.0313141;
fo=[0 20/(44.1/2) ];
des=(2-e1+e2)/2;mo=[des des];
w=[1];
h = remez(N,fo,mo,w);
nbit=8;
hs=round(h*2^nbit)/(2^nbit);
%
% hs(10)*2^8=hs(13)*2^8=-53
% hs(11)*2^8=hs(12)*2^8=-53
% cannot be expressed as three powers of two
% therefore, hs(10)=hs(13)=-54*2^(-8)
% hs(11)=hs(12)=162*2^(-8)
hs(10)=-54*2^(-8);hs(13)=hs(10);
hs(11)=162*2^(-8);hs(12)=hs(11);
figure(1)
[H,W]=zeroam(h,.0,1.,2000);
[H1,W1]=zeroam(hs,.0,1.,2000);
plot(W/pi,20*log10(abs(H)),'- -',W/pi,20*log10(abs(H1)));
axis([0 1 -90 10]);grid;
ylabel('Amplitude in dB'); xlabel('Angular frequency omega/pi');
title('Solid and dashed lines for quantized and ideal filters');
[H,W]=zeroam(h,fo(1),fo(2),2000);
[H1,W]=zeroam(hs,fo(1),fo(2),2000);
figure(2)
subplot(211)
plot(W/pi,H);grid
xlabel('Angular frequency omega/pi');
title('Passband: Ideal response H(omega)');
subplot(212)
plot(W/pi,H1-H);
grid;
ylabel('                              Zero-phase frequency response');
xlabel('Angular frequency omega/pi');
title('Passband: Quantization error E_b(omega)');
figure(3)
plot(W/pi,H,'- -',W/pi,H1);
```

```
title('Passband: Solid and dashed lines for quantized and ideal
filters');
ylabel('Zero-phase frequency response');
xlabel('Angular frequency omega/pi');
```

```
% Matlab-file half.m for plotting the responses
% for the example half-band FIR filter
% can be found in SUN's: ~ts/matlab/sldsp
clear all
close all
f(11)=162;f(10)=-54;f(9)=31;
f(8)=-21;f(7)=15;f(6)=-11;
f(5)=8;f(4)=-6;f(3)=4;
f(2)=-3;f(1)=4;
f=f*(2)^(-8);
for k=1:11
f(23-k)=f(k);end
figure(1)
[F,om]=zeroam(f,.0,2.,20000);
plot(om/pi,F);title('F(z)');
axis([0 1 0 1.05]);grid;
ylabel('F(omega)'),xlabel('omega/pi');
figure(2)
plot(om/pi,F);
title('Details for F(z) in the Passband [0, 0.907pi] ');
axis([0 0.90703 1-.03 1+.03]);grid;
ylabel('F(omega)'),xlabel('omega/pi');
x=F;
a1=2^(0)+2^(-4)+2^(-5);
a2=-2^(0)-2^(-4)-2^(-5);
a3=2^(-1)+2^(-3)+2^(-5);
a4=-2^(-3)-2^(-5)+2^(-20);
F1=x;
F2=F1.*F1;
F3=F1.*F2;
F5=F3.*F2;
F7=F5.*F2;
G=a1*F1+a2*F3+a3*F5+a4*F7;
figure(3)
plot(om/pi,G);title('G(z)');
axis([0 1 0 .55]);grid;
ylabel('G(omega)'),xlabel('omega/pi');
figure(4)
plot(om/pi,G-1/2);
title('Details for G(z) in the Passband [0, 0.907pi] ');
axis([0 0.90703 -.000001 .000001]);grid;
ylabel('G(omega) - 1/2'),xlabel('omega/pi');
```

```
figure(5)
plot(om/(2*pi),20*log10(abs(G+1/2)));title('H(z)');
axis([0 1 -160 10]);grid;
ylabel('Amplitude in dB'),xlabel('omega/pi');
figure(6)
plot(om/(2*pi),G-1/2);
title('Details for H(z) in the Passband [0, 0.4535pi] ');
axis([0 0.4535 -.000001 .000001]);grid;
ylabel('H(omega) - 1'),xlabel('omega/pi');
```

MULTIPLIER-FREE DECIMATOR ALGORITHMS FOR SUPERRESOLUTION OVERSAMPLED CONVERTERS

OUTLINE

* OVERSAMPLING AND DECIMATION

* FILTER REQUIREMENTS FOR 16/20 BIT RESOLUTION

* DECIMATOR REQUIREMENTS

* FILTER STRUCTURES & CHARACTERISTICS

* FILTER IMPLEMENTATIONS

* CONCLUSIONS

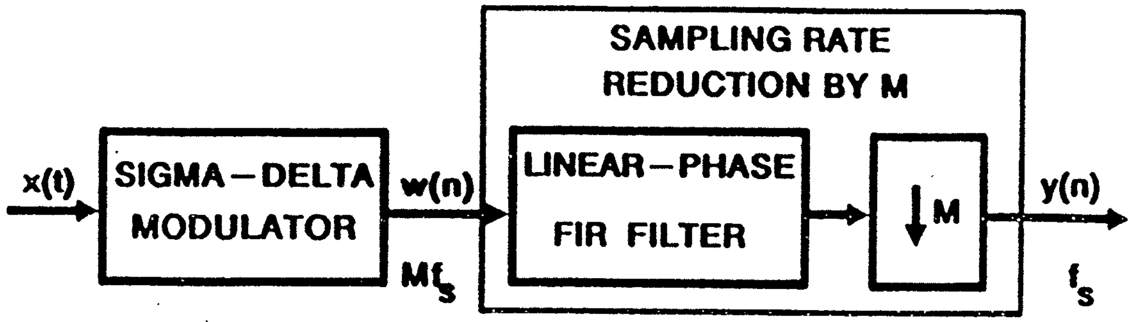
OVERSAMPLING & DECIMATION

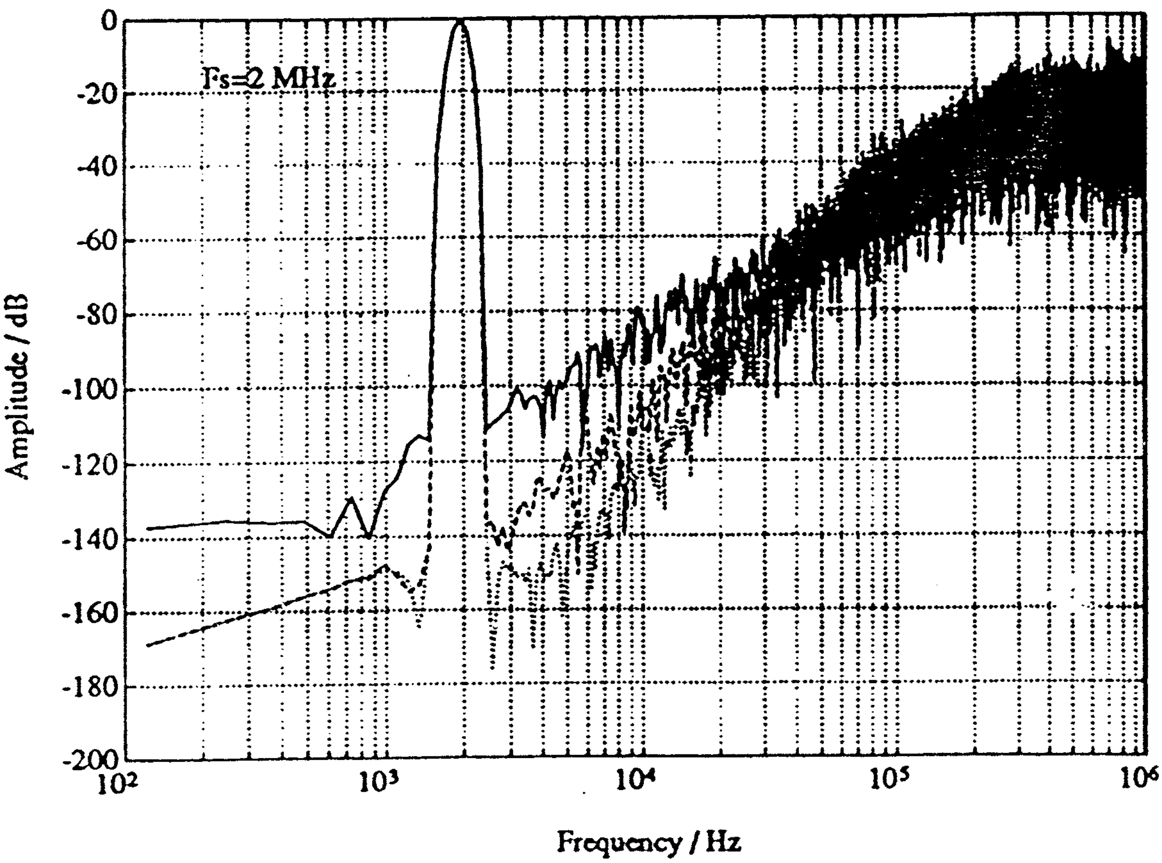
* Quantization noise attenuation

* Antialias filtering at input

* Pass band control

  -with 44.1 kHz final sampling rate band pass edge at 20 kHz



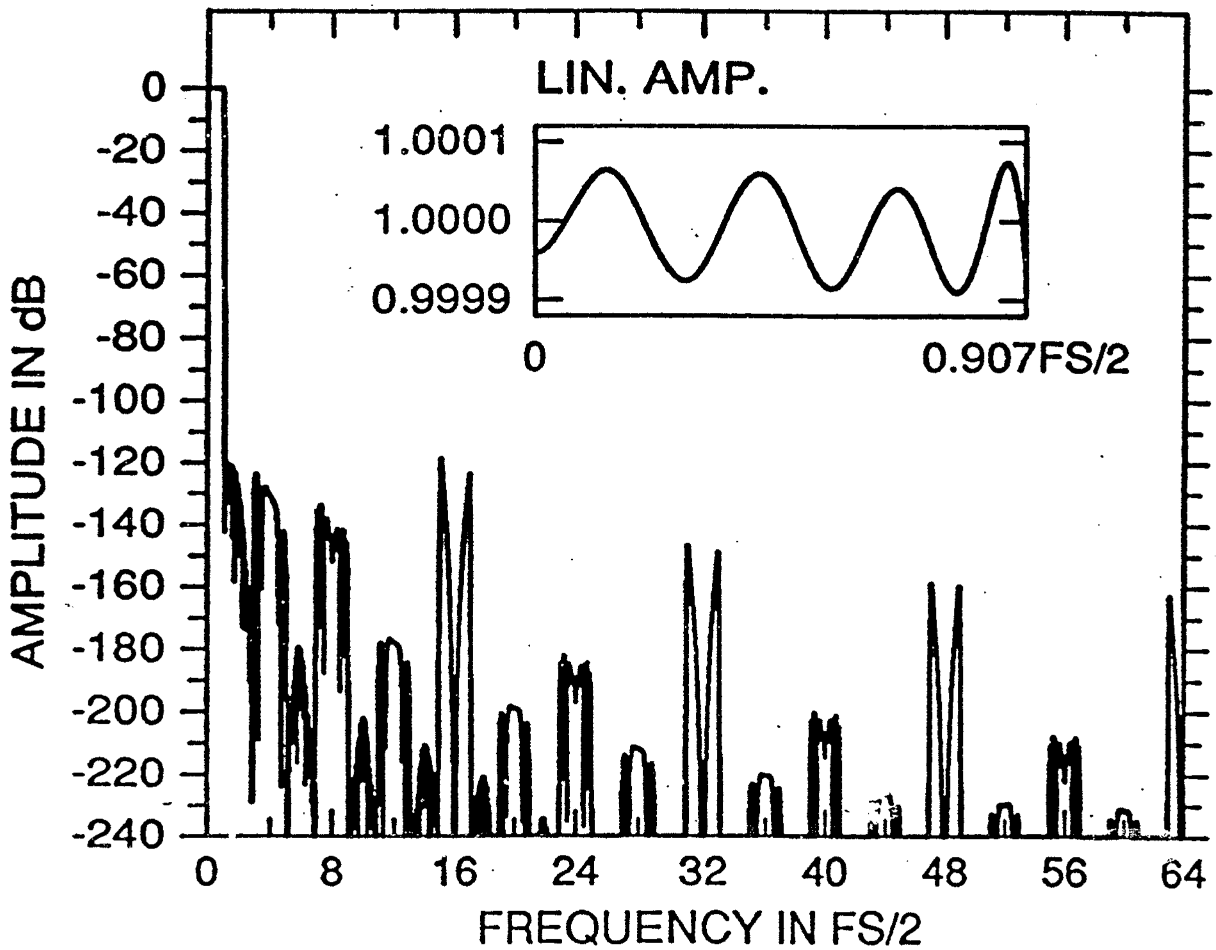


REQUIREMENTS FOR THE DECIMATOR

$$1 + \delta_p \leq |H(e^{j2\pi f/(Mf_s)})| \leq 1 - \delta_p \quad \text{for} \quad 0 \leq f \leq \alpha\frac{f_s}{2}$$

$$|H(e^{j2\pi f/(Mf_s)})| \leq \delta_s \quad \text{for} \quad (2-\alpha)\frac{f_s}{2} \leq f \leq M\frac{f_s}{2},$$

$\delta_p = 0.0001$ and $\delta_s = 0.000001$ (120-dB attenuation).



PROPOSED CLASS OF DECIMATORS

$$H(z) = H_4(z^M)H_3(z^{M/2})H_2(z^{M/4})H_1(z^{M/8})G(z),$$

$$G(z) = 2^{-P}\left[\frac{1 - z^{-4}}{1 - z^{-1}}\right]^L$$

$$K = M/8$$

$$P = L\log_2 K$$